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PENSION SIMULATION PROJECT

Public Pension Funding Practices

How These Practices Can Lead to
Significant Underfunding or
Significant Contribution
Increases When Plans Invest in
Risky Assets

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Executive Summary

Public pension funds invest more than \$3.6 trillion in stocks, bonds, and other securities with the hope of achieving an assumed investment return. If investments do as well as the funds assume or better, state and local governments can fund pensions with much less tax money than risk-free investing would require. If investments do poorly, pension funding can become precarious and government contributions may have to rise dramatically, crowding out services and infrastructure investments or requiring tax increases, and leading to calls for benefit cuts. Taxpayers, citizens who benefit from government services and investments, and workers and retirees all have a stake in the investment decisions and risks that pension funds take. Tomorrow's taxpayers, citizens, workers, and retirees also have a stake.

This report is about how the methods that public retirement systems and governments use to fund pensions interact with investment return volatility. We find that commonly used funding methods can exacerbate the risks of severe underfunding and of large increases in contributions by government employers. This is the first in a series of reports from the Rockefeller Institute of Government's Pension Simulation Project. Later reports will examine other facets of these risks and how governments and plans may address them.

Governments make contributions to pension plans with the goal of having enough money to pay future benefits when due. Governments and pension funds cannot predict the future with certainty, so they ordinarily adjust contributions as they go — paying more if things aren't working out as well as expected, or paying less if things are working out better than expected.

The biggest uncertainty is how well the pension plan's investments will do. From the 1970s through the mid-1990s, it was possible to earn 7 to 8 percent without taking much risk because U.S. Treasury securities, which are virtually risk-free, paid that much. Now, with Treasury rates much lower, public pension funds can only earn about 2 to 3 percent without taking risk. As a result, they must either invest in riskier assets or lower their assumed returns. For the most part they have chosen the former, continuing to assume they will earn 7 to 8 percent by investing in riskier assets, and assuming that things will work out over the long run. Investment gains and losses can thus be much greater now than in earlier years.

If a pension fund's investments fall short in a given year, the government could shore up the fund by making a large extra contribution. This contribution would have the advantage of restoring the fund immediately, protecting beneficiaries, and not pushing the costs for yesterday's services off to tomorrow's taxpayers. But this practice could be devastating for governments if returns fall far short of assumptions, leading to sharp temporary cuts in budgets and public services or to tax increases. No large

pension plan is funded this way. Instead, governments and pension plans use funding policies and practices designed to stretch out repayment of shortfalls over longer periods, in a smoother fashion, to make them easier to absorb in budgets. These practices work similarly for positive investment surprises.

How long it takes to repay shortfalls or to reflect good news depends primarily on three elements: (1) the “discount rate” the plan uses to estimate the cost of future benefits in today’s dollars; (2) the method used to amortize shortfalls and overages (i.e., how the plan determines the amount to set aside each year); and (3) when and how the plan recognizes investment gains and losses, known as asset smoothing.

Funding policies and practices that take a long time to repay shortfalls protect current taxpayers and beneficiaries of government services from sharp and possibly unaffordable changes. But they create risk that the pension plan will become deeply underfunded and that future taxpayers who never benefited from past services will have to pay for them. This is particularly true if the plan suffers a series of shortfalls over several years. Funding policies and practices that repay shortfalls quickly shore up the fund and protect future taxpayers, but are risky for current taxpayers and government service beneficiaries.

Governments and plans commonly use methods that stretch repayments out over long periods — often for thirty years, or sometimes forever through use of “open” methods that constantly extend the period. About two-thirds of the unfunded liability of public pension funds is being repaid using methods that stretch repayments out for thirty years or more.

We have developed a model to evaluate pension risks under different funding policies and investment return scenarios. *We find that while the most-common funding policies and practices reduce contribution volatility, they increase the likelihood of significant underfunding.* Funding policies are unlikely to bring underfunded plans to full funding within thirty years, even if investment-return assumptions are met every single year and employers make full actuarially determined contributions. For example, a fairly typical plan — a 75 percent funded plan that amortizes gains and losses as a constant percentage of payroll over a constantly extended thirty-year “open” period, with five-year asset smoothing — would reach only 85 percent funding after thirty years even if it earned its assumed 7.5 percent every year.

However, plans rarely achieve their assumptions exactly in any given year. If investment returns vary from year to year — as they generally do — the situation is much more difficult, even if expected investment returns are correct on average. According to our model, the same typical plan as above would have a 17 percent chance of falling below 40 percent funding over the next thirty years if its investment return assumption is correct on average but returns vary in plausible ways (measured with a 12

percent standard deviation). In other words, the plan would have a one in six chance over the next thirty years of falling into the sort of crisis territory currently inhabited by major pension funds in Illinois and Kentucky, *even if expected return assumptions are correct on average and governments pay full contributions*. If sponsors do not pay full actuarial contributions, or if reasonable expectations for investment returns are less than 7.5 percent, the risk of severe underfunding would be even greater.

We find that the risks of severe underfunding vary dramatically with the choice of funding method. If, instead of amortizing over a thirty-year open period, the plan uses a closed period that ensures repayment after thirty years, the risk of severe underfunding falls to 10 percent. And if it shortens the period to fifteen years or pays a fixed dollar amount rather than amounts that start low and rise with payroll and does not use asset smoothing, the risk falls to four percent or less. In practice many governments do not always pay full actuarially determined contributions, which greatly exacerbates the risks of severe underfunding.

The risk of severe underfunding increases over time. It is often said that pension funds are very long-term investors and can achieve their financial goals by riding out market fluctuations. While differences from expected returns are likely to narrow over the long run, those differences are compounded over many more years, so that fluctuations in assets available to pay pension benefits actually increase with time.

It is clear that current funding policy is inadequate, particularly in light of pension plans' investments in stocks and other assets with volatile returns. The risks to governments and pension plans in the current environment are greater than commonly understood.

There is no easy way out. Pension plans can de-risk, reducing the volatility of their investment returns and reducing the volatility of contributions. However, reducing risk almost certainly will require lowering earnings assumptions, which will drive up contribution demands from governments and crowd out services or require tax increases.



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PENSION SIMULATION PROJECT

Public Pension Funding Practices

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Introduction

Public defined benefit pension funds provide retirement income for nearly ten million Americans. Although the sharp stock market declines of the Great Recession are more than seven years in the past, public pension plans remain underfunded by approximately \$1.7 trillion despite contribution increases,^{1,2} in part because the methods state and local governments use to address funding shortfalls typically stretch repayments out over long periods of time.

Defined benefit pensions are deferred compensation: An employee provides services now, and the employer promises to pay compensation during retirement. Prefunding these promises by setting aside funds in the present that, together with investment earnings, will be sufficient to pay benefits when due helps achieve intergenerational equity. It ensures that current taxpayers pay the full cost of current services, and it helps achieve benefit security by ensuring that funds will be available for promised pensions at the time benefits must be paid.³ Although some economists argue that 100 percent funding many not be an optimal goal because governments may enhance benefits when plans become overfunded, we assume that full funding is the proper goal.⁴

Prefunding requires the sponsoring government to make contributions each year, and funding policy is the set of methods and rules used to determine those contributions. As we use the term, “funding policy” is broader than the actuarial cost method used to allocate benefits to past and future service. It also includes when and how shortfalls and gains are recognized and reflected in

contributions. Several organizations have proposed principles for funding policy. Recently, a panel commissioned by the Society of Actuaries (SOA) set out the following principles:⁵

- **Adequacy:** Contributions, together with investment income, should be sufficient to pay promised benefits as they come due, under a range of economic conditions.
- **Intergenerational equity:** Achieving intergenerational equity requires paying the costs of pension benefits over employees' working lifetimes.
- **Cost stability and predictability:** All else equal, government officials generally prefer predictable contributions that fit well into state and local budgets.

In practice, funding methods and rules consist of a discount rate used to value liabilities, amortization rules for taking unexpected gains and losses into account, and asset smoothing rules to determine when and how swings in the market values of assets are reflected in the calculations. These are the main elements needed to calculate actuarially determined contributions. In addition, the contribution behavior of governments — the choice about whether to pay actuarially determined contributions and the extent to which they will be paid — is often considered part of funding policy.

Each of these elements affects the goals of adequacy, intergenerational equity, and cost stability. If investment returns are highly variable, a plan's funded status and the employer's contributions may be subject to large swings that depend in part on these elements of funding policy. Large investment shortfalls can lead to severe underfunding, and to large but lagged increases in actuarially determined contributions. Underfunding and contribution increases can stress the political system, leading to cuts in public services, tax increases, budget gimmicks, and calls for benefit cuts. Sharp investment gains can lead to the opposite.

In this report, we examine how investment return variability interacts with funding policy to create the risk of severe underfunding and large increases in employer contributions. We do this using a "stochastic" model that calculates the annual finances of a pension fund under 1,000 (or more) sets of investment returns, under different funding policies.

Funding Policy in Practice

Funding policy begins by estimating how much will be owed to retirees at the date of retirement, reflecting the plan's benefits, vesting rules, assumptions about mortality, and other factors. While this requires assumptions that are difficult to estimate, it is conceptually straightforward.

The next step is to estimate how much of those benefits have been earned by a given date before retirement. This is more difficult conceptually, because benefits under final-salary pension plans accrue at much higher rates late in a worker's career than early in the career, because salaries and years of service are higher

later in a worker's career. If governments were to fund benefits as they accrue, contributions for individual workers would rise sharply during their careers. In part for these reasons, actuaries have developed methods that take into account projected increases in salaries and require greater contributions earlier in the career. The most common such approach, entry age normal, includes projected salary growth in the estimate of future benefit calculations, and spreads projected costs over the employee's working career in proportion to projected salaries. In short, contributions will rise at the rate of payroll growth, as long as all other assumptions are met. By providing a gradual path for contributions, this approach can satisfy the cost stability principle. Keeping costs in proportion to payroll and having benefits fully funded by the end of a worker's career is consistent with the principle of intergenerational equity as well.

We discuss the major elements of funding policy below.

Discount Rate

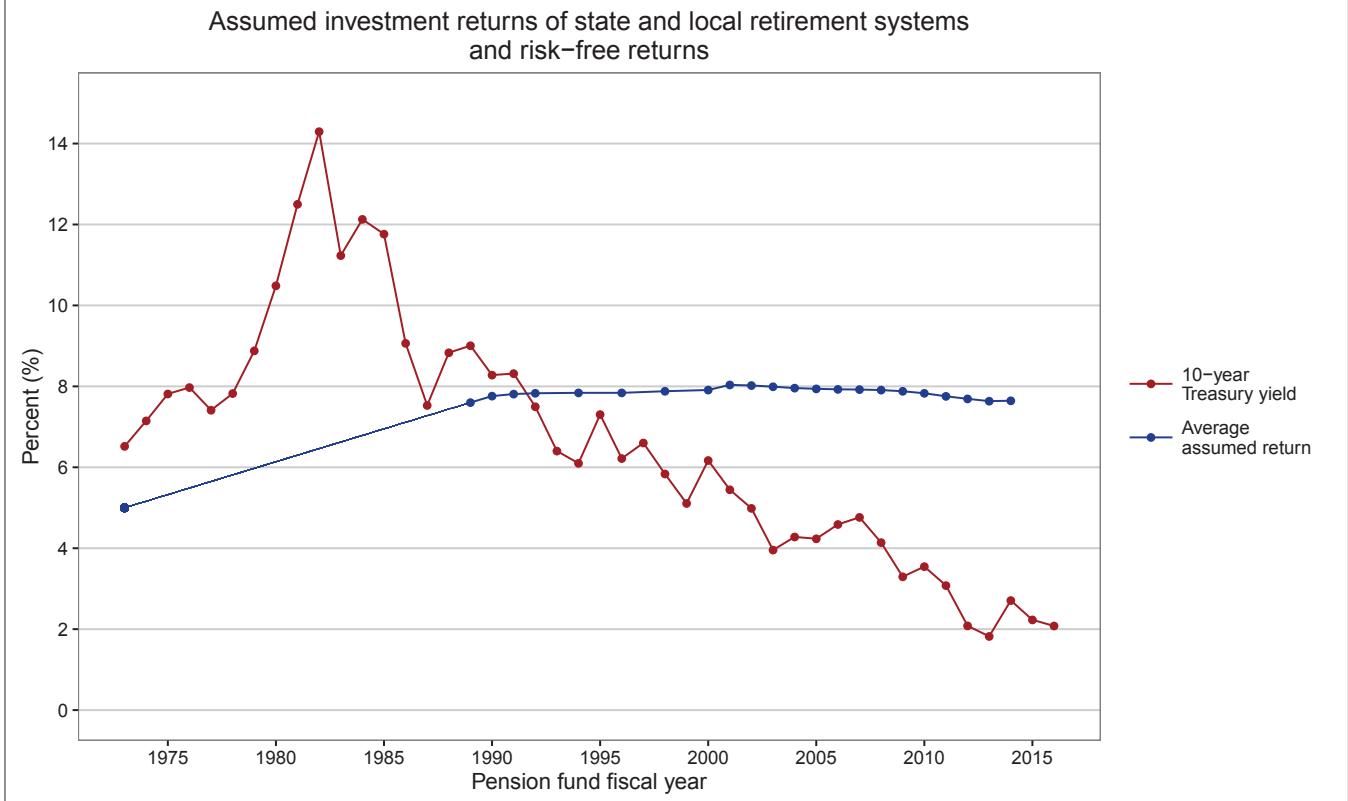
At what rate should future benefits be discounted — that is, converted to present day dollars? Economists have concluded that benefits should be discounted at rates that reflect the riskiness (or intended riskiness) of their cash flows; if pension benefits almost certainly will be paid, that suggests using rates based upon risk-free or nearly risk-free assets.⁶ There are some minor disagreements among economists about how, exactly, to translate this into specific market-related rates, but there is broad agreement on the larger issue. Valuing liabilities at risk-free rates does not mean funds should invest only in risk-free assets; that is an entirely separate decision. By contrast, actuaries discount benefits based on the rate they expect will be earned on the pension fund's asset portfolio, and that expectation is often based on historical returns rather than current market conditions.⁷

Risk-free returns have fallen substantially over the past three decades. They are lower than expected returns on a diversified portfolio of investments, and are far lower than the earnings assumed by the median pension fund, currently about 7.6 percent.⁸ While public pension funds have lowered earnings assumptions slightly, they have not come down as much as risk-free returns have declined. (See Figure 1.)⁹

The choice of discount rate is a technical issue with large consequences. The lower the discount rate, the higher the estimate of liability. One academic paper concluded that liabilities of state-run pension plans for the U.S. as a whole were more than \$2 trillion higher when discounted using risk-free rates.¹⁰

Estimated annual costs are higher, too, when discount rates are lower. In one simple example, the annual "normal cost" of benefits is nearly twice as great when valued at a 6 percent discount rate as at an 8 percent rate, and nearly four times as great when valued at a 4 percent rate.¹¹ If benefits were valued using risk-free or nearly risk-free rates, normal costs would be several

Figure 1. Risk-Free Returns Have Declined Substantially But Assumed Returns of Public Pensions Have Not



multiples of actuaries’ current estimates. The Bureau of Economic Analysis has estimated that annual normal costs calculated at a 5 percent discount rate were \$89 billion higher than actual employer contributions in 2013.¹² Funding these costs at risk-free rates, which are lower than 5 percent, would have required contributions to be higher still.¹³ The higher contributions that governments would have to pay if benefits were discounted at risk-free rates can be thought of in two ways: as an indication of how much money public pension plans believe they can save their governments through successful investing, and as an indication of the magnitude of risk that pension fund investments entail.

To fund benefits with complete security – that is, with no risk of shortfalls and with no chance of pushing costs off to the future – a pension fund would calculate contributions using a risk-free rate. Given the huge increases in government contributions that would be required, that certainly will not happen anytime soon.

Amortization Methods and Periods

In addition to the discount rate, funding policies must take into account amortization, or how to “stretch out” repayment. Common amortization methods vary primarily along three dimensions:

1. **Closed or open:** Closed methods use a fixed period over which to amortize liabilities – for example, thirty years – after which the liability is completely paid off. By contrast, open methods continually reset the length

of the period. Under an open thirty-year amortization method, the first payment would be calculated based on a thirty-year period, but the second payment would be based on a new thirty-year period (rather than twenty-nine years), after adjusting the unpaid balance to take into account the prior payment. Closed methods pay off liabilities more quickly than open methods. In fact, open methods never completely pay down a liability, although the liability may be reduced substantially.

2. **Level dollar or level percent of payroll:** Level dollar methods calculate a fixed annual dollar payment, similar to a home mortgage and to the way that governments ordinarily repay bonds issued to finance infrastructure. By contrast level-percent methods calculate a fixed percentage of each year's payroll. Under the level-percent method, the initial payment is lower than later payments, and payments are expected to grow each year at the same rate as payroll grows. (Payments could decline if payroll were expected to decline, but in practice payroll usually is assumed to grow.) Level dollar methods pay off liabilities more quickly than level-percent methods.
3. **Length of amortization period:** The longer the amortization period, the lower the annual payments and the longer it will take to pay off the liability.

Neither the Governmental Accounting Standards Board (GASB) nor the Actuarial Standards Board (ASB) has authority to mandate amortization methods, but their standards and pronouncements do have influence. Until recently, GASB Statement 25 governed accounting for public pension funds and it established a maximum amortization period of thirty years for accounting.^{14,15} The successor to GASB 25, GASB 67, requires an amortization period for financial reporting purposes that is based on the remaining service life of all employees, although plans are not required to use this for funding purposes.¹⁶ In many cases this is much shorter than thirty years.

Table 1 shows the distribution in 2013 of major public pension plans along these three dimensions, based upon amortization methods described in pension plan financial reports as entered into the Center for Retirement Research's Public Plans Database; these financial-report methods are likely to correspond quite well to methods used for funding purposes.¹⁷ The table summarizes data for the 138 out of 160 plans for which there were sufficient data to classify amortization methods. The percentages given in the text below are based on plans with available data.

Nearly 40 percent of the plans that could be classified used open methods, with their infinite repayment periods, and about 60 percent used closed methods. This table may overstate the use of closed methods because some hybrid methods are difficult to classify. For example, for 2012 the Missouri Local Government Employees

Table 1. Prevalence of Amortization Methods Among Major Public Pension Plans

Amortization methods and periods, 2013

Amortization method	Number of plans	% of plans	% of unfunded liability	Remaining amortization periods for plans with data			
				Number of plans with available data	Up to 15 years	16 to 29 years	30 or more years
<i>Open methods (longer repayment)</i>	54	39%	57%	51	1	12	38
Level percent - open	41	30%	46%	38	0	10	28
Level dollar - open	13	9%	11%	13	1	2	10
<i>Closed methods (shorter repayment)</i>	84	61%	43%	78	12	46	20
Level percent - closed	56	41%	26%	54	7	33	14
Level dollar - closed	28	20%	16%	24	5	13	6
Total for plans with detailed data	138	100%	100%	129	13	58	58

Note: Details may not add to totals due to rounding.

Source: Rockefeller Institute analysis of the Public Plans Database, Center for Retirement Research, Boston College

Retirement system is classified in the Public Plans Database as using closed amortization. However, according to its comprehensive annual financial report, “upon attainment of 15 years liabilities are amortized over an open 15-year period by level percent of payroll contribution.”¹⁸ Its amortization method behaves like an open method and the liability is never fully paid down. Additionally, many plans using closed amortization reset the amortization period about halfway through, when payments are rising.¹⁹ Because of these classification difficulties, pure closed amortization is not as common as the table suggests.

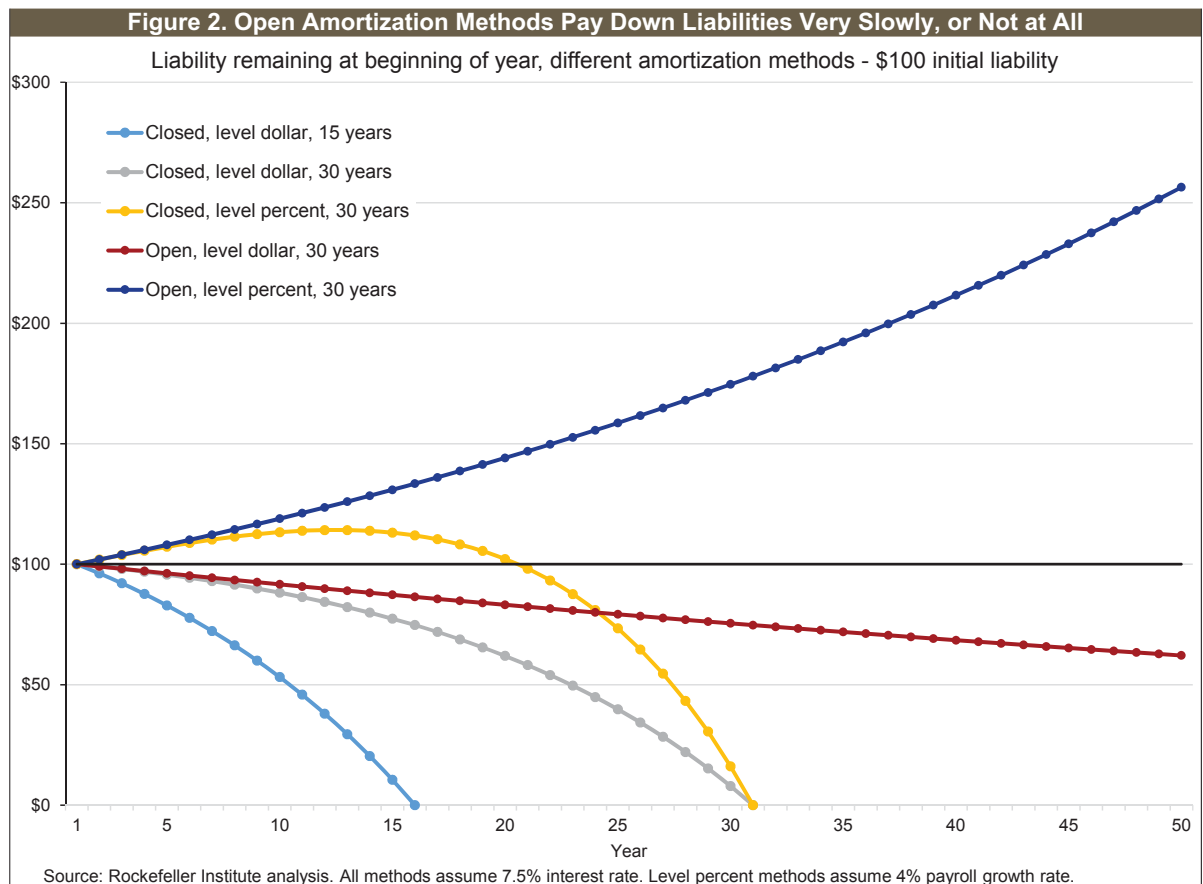
Level-percent amortization methods, which have lower early payments and higher later payments than level dollar methods, were more common than the level dollar methods. Seventy-one percent of plans, with 72 percent of unfunded liability, used level-percent methods (calculated from Table 1). Those using the longest repayment method, level-percent open, accounted for the greatest share of unfunded liability (46 percent).

Most plans have long remaining amortization periods. In total, 45 percent of plans had remaining amortization periods of thirty years or more, and another 45 percent used periods of sixteen to twenty-nine years.²⁰ According to separate calculations not shown in the table, about two-thirds of the unfunded liability of public pension funds is being repaid using methods that stretch repayments out for thirty years or more. Amortization periods were the longest for the open methods, where 75 percent of plans used periods of thirty or more years (in most cases they used exactly thirty years). Put differently, the method that generally takes the longest to pay down an unfunded liability (open) tended to be combined with the longest amortization periods, extending the period to pay down liability.

In the last decade, plans have adopted slightly more aggressive repayment schedules. However, most pension funds, and those with the largest unfunded liabilities, still use methods that stretch repayments out substantially. Fifteen-year closed level-dollar amortization is about the most conservative amortization method used by large plans, and it is very rare. In 2013 only six of the thirty-seven plans in the Public Plans Database that used level-dollar amortization and had amortization-period data used an amortization period of fifteen years or less. At the other extreme, thirty-year open level-percent amortization is quite common. In 2013, twenty-eight large plans accounting for 42 percent of the unfunded liability of plans with available data used open level-percent with an amortization period of thirty or more years.

Level-percent methods, with their lower initial payments, are attractive to governments that sponsor public pension plans. The lower initial payments allow them to keep taxes lower or services higher in early years after investment shortfalls. However, low *initial* payments come at the expense of greater contribution payments later and greater tax and service trade-offs later. The same is true when investment returns or other actuarial factors work out better than expected: level-percent methods defer more of the good news than do other methods.

Figure 2 illustrates the amortization of an unfunded liability of \$100 under five combinations of amortization elements, ranging



from fast-amortizing fifteen-year closed level dollar to never fully amortizing thirty-year open level percent. The three closed methods pay off the liability eventually and thus each crosses the horizontal axis after the fifteenth or thirtieth year, respectively.

There is a horizontal line at the \$100 mark. If a line for a particular method goes above this level, then the liability has actually risen above the original liability, which happens for both the closed and open level-percent methods. Rising above the original liability, known as negative amortization, has been widely criticized. With closed level-percent amortization, the liability is eventually paid off. However, under open level-percent amortization, not only is the liability never paid off, it rises forever in nominal dollars under typical assumptions. The accounting standard that allowed negative amortization was adopted over the dissent of the GASB chair, who argued that this was not an amortization method at all.²¹

Although nominal payments and liability can rise forever under thirty-year open level-percent amortization, liabilities shrink as a share of payroll as the time horizon lengthens, if payroll grows faster than liability and other assumptions are met. After fifty years, nominal liability under thirty-year open level-percent amortization is more than twice its initial value and continues to rise, but relative to payroll it is approximately 40 percent of its original level. Contributions as a percentage of payroll also decline. Although the burden of amortization contributions falls, it continues forever — long after working careers and even retirement years have ended for the people for whom that liability was incurred.

Asset Smoothing

The third major element of funding policy is asset smoothing, or when and how to recognize investment gains and losses. As with amortization of unfunded liability, asset smoothing imposes greater stability on pension contributions in light of investment return volatility. Asset smoothing works by recognizing recent investment gains or losses incrementally over several years. Actuaries construct the “actuarial value of assets,” which reflects the extent to which investment gains and losses have been recognized. One simple form of asset smoothing phases in unexpected gains or losses over five years: for example, if a pension plan expected to earn \$900 in a given year but only earned \$400, it would spread the \$500 shortfall over five years, only recognizing \$100 in the first year, and \$200 in the second year, and so on. Actuarial assets can be less than or greater than the market value of assets.

Almost all plans use some form of asset smoothing — between 2001 and 2013, out of 150 plans in the Public Plans Database all but a handful (between five and nine in any year), used some form of asset smoothing. The vast majority used five-year averaging of asset values. Seven plans used ten-year smoothing in 2012. Asset smoothing often is accompanied by caps and collars, which

limit the divergence between actuarial value of assets and market value of assets.

Asset smoothing has a very different effect than amortization: It forestalls sharp contribution increases or decreases in the first few years after investment losses or gains by simply not recognizing those gains or losses immediately and fully. By contrast, amortization spreads actuarial gains or losses, once recognized, over relatively long time periods.

After an investment shortfall, initial amortization payments under asset smoothing are much lower than they otherwise would be because only a portion of the loss is recognized initially, but payments ramp up after five years. With closed amortization, payments stay higher than they otherwise would be until the end of the amortization period, after which contributions fall sharply for five years.

Asset smoothing can be attractive to elected officials or others focused on the near term. By creating a ramp to a new level of contributions, it provides time for financial and political planning. If government tax revenue is cyclical in a way that coincides with investment return cycles, asset smoothing could defer contribution increases to periods when government tax revenues have recovered from cyclical declines. However, by insulating elected officials with short time horizons from the near-term consequences of investment risk — risk that their successors may have to bear — asset smoothing creates what economists call a moral hazard, where one party takes risks and another bears the costs.

Adjustments to Contribution Policies

Governments and pension plans use many variants of the smoothing methods described above. One approach that is potentially damaging to pension funding and pension benefit security is statutory rules that allow or require governments to pay less than actuarially determined contributions; if contributions are below what actuaries request, the plan may never be on a path to full funding. This is the primary cause of underfunding in the most deeply underfunded plans. According to a recent analysis of 110 large state-administered plans over the 2001-10 period, only 50 percent of the observations had fully actuarially determined contributions that were not overridden by explicit contributions in statute (e.g., a fixed percentage of payroll), or limited or capped in some way, or overridden in appropriations bills.^{22,23}

Prior Work on Stochastic Simulation of Public Pension Funds

One of the main reasons that public pension funds use smoothing policies is to avoid sharp increases or decreases in sponsor contributions as a result of uncertain investment returns. Thus, to truly understand the effects of these policies, researchers must take into account the potential variability, or “stochastic” nature, of investment returns. While several academics and

practitioners have constructed stochastic simulation models of pension funds,^{24,25} we are aware of only two significant studies that have used stochastic simulation models to examine the relationship between volatile investment returns and pension fund contributions.

The Center for Retirement Research found that “combining percent of pay with an open 30-year amortization schedule produces amortization payments that are inadequate to fund the system within 30 years” even if investment return assumptions are met.²⁶ They also found that full funding could not be achieved if plan sponsors paid only 80 percent of required contributions.

American Enterprise Institute researcher Andrew Biggs examined a “steady state” pension fund and concluded that common funding practices resulted in higher average costs when investment returns vary than under the usual actuarial assumption of the same investment return every year. He also concluded that “If public plans wish to achieve both contribution stability and intergenerational equity, they will simply have to pay more and take less investment risk.”²⁷

Our findings are consistent with previous work and extend the analysis over additional funding policies and investment scenarios.

Data and Methods

The Simulation Model

To examine the interplay between stochastic investment returns and funding policies, we have constructed a stochastic simulation model of public pension plans. The model differs from prior work in that it allows us to examine the year-by-year dynamics of pension fund finances for plans with real-world characteristics, under different investment return scenarios and different funding policies. Starting from an initial position (e.g., 75 percent funded), it projects the future annual assets and cash flows, including benefit payments, employer and employee contributions, and investment income, based upon given model inputs.

The most important model inputs include:

- *Retirement benefit rules*, including the benefit multiplier per year of service, vesting rules, allowable retirement ages, and annual benefit percentage increase, if any (we do not call this a COLA, or cost-of-living-adjustment, because it does not depend on economic conditions).
- *Plan demographics in the initial year* including number of workers by age and entry age and their average salaries; number of retirees by age and their average benefit; and projected annual growth in the workforce.
- *Decrement tables* with mortality rates, retirement rates, and separation rates.

- *Salary schedules* that define how worker salaries are expected to change over time and with experience.
- *Inflation and aggregate payroll growth assumptions.*
- *Actuarial rules and methods for determining actuarial liability, normal cost, and an actuarially determined contribution.* These include the actuarial cost method (e.g., entry age normal), discount rate (which can be different from assumed and actual investment returns), asset-smoothing rules if any, and amortization rules (open or closed, level percent or level dollar, and length of amortization period).
- *Information to determine employee and employer contributions.* For employee contributions, this is a fixed percentage of payroll. For employer contributions, this defines whether the employer pays the actuarially determined contribution, or pays according to some other rule such as a fixed percentage of payroll.
- *Rules or data specifying investment returns:* Investment returns can be deterministic or stochastic.
 - A deterministic run might have a single investment return applicable to all years (e.g., 7.5 percent per year) or it might have a set of deterministic returns, one per year (e.g., 10 percent for each of the first twenty years, followed by 5 percent for each of the next twenty years). When investment returns are deterministic, we only run a single simulation since results will not vary from run to run.
 - A stochastic run might draw investment returns randomly each year from a probability distribution – for example, from a normal distribution with a 7.5 percent mean return and a 12 percent standard deviation. (More complex investment return scenarios are possible, too.) When we run the model with stochastic investment returns, typically we conduct 1,000 simulations for a given set of inputs, so that we can examine the distribution of results.

The model can be used to examine prototypical pension funds, or can be used with data for actual pension funds. In the analysis that follows, we use a prototypical fund that resembles real-world pension plans in important ways. It has a typical age distribution of workers and retirees, and benefits generally are calculated as 2.2 percentage points per year of service multiplied by the average of the final three years of salary, plus a 2 percent annual increase in benefits.²⁸ The age structure of the plan population is based on our analysis of data in the Public Plans Database, and is similar to the population of the Arizona Public Employees Retirement System, which we found to be fairly typical in many ways. The plan has new hires each year sufficient to keep the number of active workers stable. The plan sponsor makes contributions each year

that, when added to a 5 percent employee contribution, satisfies the actuarially determined contribution. In the analysis below the plan starts out with a 75 percent funded ratio, broadly consistent with the median 72 percent funded ratio in the Public Plans Database in 2013. (We also use our model to examine fully funded plans and plans with other funding levels, but do not describe the results here, except where they provide further insight.)

We assume that investment returns follow the normal distribution, with a mean long-run compound return of 7.5 percent and a standard deviation of 12 percent. The mean is consistent with what the typical plan assumes today. (See [Arithmetic and Geometric Mean Investment Returns](#) in the appendix for a discussion of arithmetic and geometric mean returns.) The standard deviation is broadly consistent with our review of simulations and investment return analyses performed elsewhere.²⁹ A normal distribution with a standard deviation of 12 percent means that, in a typical year, the pension fund has a one in six chance of falling at least 12 percentage points short of its investment return assumption and a one in six chance of exceeding its investment return assumption by at least 12 percentage points — the chance of rolling any single number with a fair six-sided die. With approximately \$3.6 trillion of public pension defined benefit plan assets under investment, a 12 percent single-year investment return shortfall is equivalent to more than \$425 billion for the United States as a whole.

Investment returns are assumed to be independent of each other from year to year — bad investment years are not necessarily followed by good investment years, and vice versa. In later work under this project, we will examine variants in which investment returns may be correlated over time.

Because investment returns are random in the model, we might obtain virtually any sequence of returns in a single run of the model (which we call an individual simulation), but if we run enough simulations, on average the results will reflect our assumed distribution of returns (i.e., a mean compound annual return of 7.5 percent and a standard deviation of 12 percent). We run the model 1,000 times to gain insight into the likely distribution of outcomes.

Funding Policies That We Simulate

In the analysis below, we simulate funding policies that cover the range of existing practices, plus one new proposal. We examine the following combinations of policies, which range approximately from fastest repayment of underfunding to longest repayment.

- Fifteen-year closed amortization, with level dollar and alternatively with level-percent amortization.
- Thirty-year closed amortization, with level dollar and level-percent amortization.

- Thirty-year closed amortization, level-percent amortization, and five-year asset smoothing.
- Each of the above methods, with open amortization.

In addition, we examine the Society of Actuaries' proposed standardized contribution benchmark, a comparison measure that combines fifteen-year open level-percent amortization and five-year asset smoothing with a market-based discount rate.³⁰ For details see the appendix section, [The SOA Blue Ribbon Panel's Standardized Contribution Benchmark](#).

Measures We Use to Evaluate Results

We are primarily concerned about two kinds of risks:

- Extremely low funded ratios, which create a risk to pension plans and their beneficiaries, and create political risks that could lead to benefit cuts; and
- Extremely high contributions, or large increases in contributions in short periods of time, which pose direct risks to governments and their stakeholders, and in turn could pose risks to pension plans and their beneficiaries.

We focus on downside risks because they have the potential to get plans and governments into trouble. There are upside risks as well.

There usually are trade-offs between these two kinds of risks. If a pension plan has a contribution policy designed to pay down unfunded liabilities very quickly, it is unlikely to have low funded ratios, but it may have high contributions. If a pension plan has a contribution policy designed to keep contributions stable and low, there is greater risk that funded ratios may become very low because contributions may not increase rapidly in response to adverse experience.

We use three primary measures to evaluate these risks.

Probability That the Funded Ratio Will Fall Below 40 Percent During the First Thirty Years

When returns are stochastic, many outcomes are possible, including very extreme outcomes, so it does not make sense to focus on the worst outcomes or the best outcomes. We are particularly concerned about the risk of bad outcomes, and one useful measure is the probability that the funded ratio, using the market value of assets, will fall below 40 percent in a given time period.

We choose 40 percent because it is a good indicator of a deeply troubled pension fund. In 2013, only four plans out of 160 in the Center for Retirement Research's Public Plans Database had a funded ratio below 40 percent — the Chicago Municipal Employees and Chicago Police plans, the Illinois State Employees Retirement System, and the Kentucky Employees Retirement System. Each plan is widely recognized as being in deep trouble, with the likelihood of either substantial tax increases, service cuts, or benefit cuts yet to come.

In the scenarios that follow, plans start out with a 75 percent funded ratio. Falling to 40 percent funded in the first year would require an investment shortfall of well over 40 percent, which is not likely in a single year. But as the time period extends, there is a chance of an extended period of low returns, leading to severe underfunding. This measure evaluates the likelihood of this occurring.

Probability That Employer Contributions Will Rise Above 30 Percent of Payroll During the First Thirty Years

Extremely high contributions can create great political and financial pressure on plan sponsors and may lead to benefit cuts, tax increases, and crowding out of expenditures on other public services. We use the probability that the employer contribution will rise above 30 percent of payroll as of a given year to evaluate how likely it is that the plan sponsor may face the pressure of high contributions. As the time period extends and the chance of a long period of low returns rises, the probability of having a high employer contribution anytime in that period will increase.

Probability That Employer Contributions Will Rise By More Than 10 Percent of Payroll in a Five-Year Period

Making contributions stable and predictable is one of the most important goals of funding policies from the perspective of the employer. Sharp increases in employer contributions, even if not large enough to threaten affordability, can cause trouble in budget planning. We use the probability that the employer contribution will rise by more than 10 percentage points of payroll in a five-year period to measure this possibility. In some of the policies we examine, extremely low returns in a very short time period as may occur in a severe financial crisis may push up the required contribution considerably even after being dampened by asset smoothing and amortization policies.

Results

An individual simulation entails a sequence of annual pension fund finances for a single fund, under a particular funding policy, with a single sequence of annual investment returns chosen randomly. To illustrate how the model works and the information it produces, we begin by examining three arbitrarily chosen individual simulations. We then summarize results for 1,000 simulations. The advantage of examining individual simulations is that they demonstrate the volatility that a pension fund might experience over time in a way that is hard to see with summary measures. The disadvantage is that we cannot generalize from a single run because virtually any sequence of investment returns is possible if they are chosen randomly.

Illustrative Simulations

In our illustrative simulations we examine a single funding policy: thirty-year level-percent open with five-year asset

smoothing, a common approach as discussed earlier. The three individual simulations are:

- A “deterministic” run in which the pension fund earns exactly 7.5 percent on its investments every year.
- A run with generally high investment returns in the early years and lower investment returns in later years (sim #56). This run results in a compound annual return of 7.5 percent over the first thirty years.
- A run with generally low investment returns in the early years and higher investment returns in later years, also resulting in a compound annual return of 7.5 percent over the first thirty years (sim #228).

Figure 3 shows the annual return and cumulative compound return for each of the two simulations. As is apparent, even though these two individual simulations achieve the assumed 7.5 percent compound return by the end of thirty years, *annual returns are extremely volatile even when investment return assumptions are attained*. Furthermore, these two selected simulations understate the volatility that plans face: most of the 1,000 simulations do NOT result in a compound average return of 7.5 percent over thirty years – the thirty-year average falls below 6 percent about one quarter of the time, above 9 percent about one quarter of the time, and between 6 percent and 9 percent half of the time. Thus, contributions and funded ratios will be more variable most of the time than in the illustrative simulations we examine below. We summarize the full range of variability later.

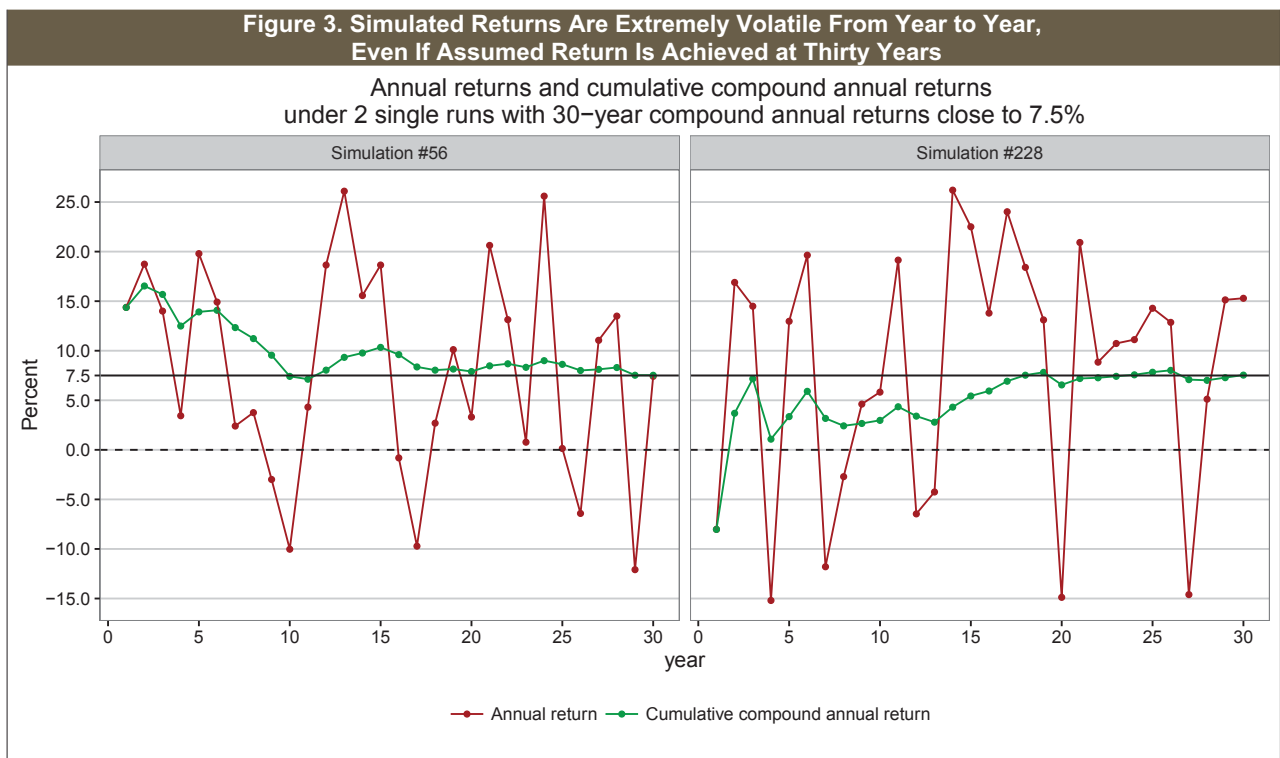


Figure 4. Employer Contribution Rates Vary Dramatically Even for Simulations That Have the Same Compound Average Return

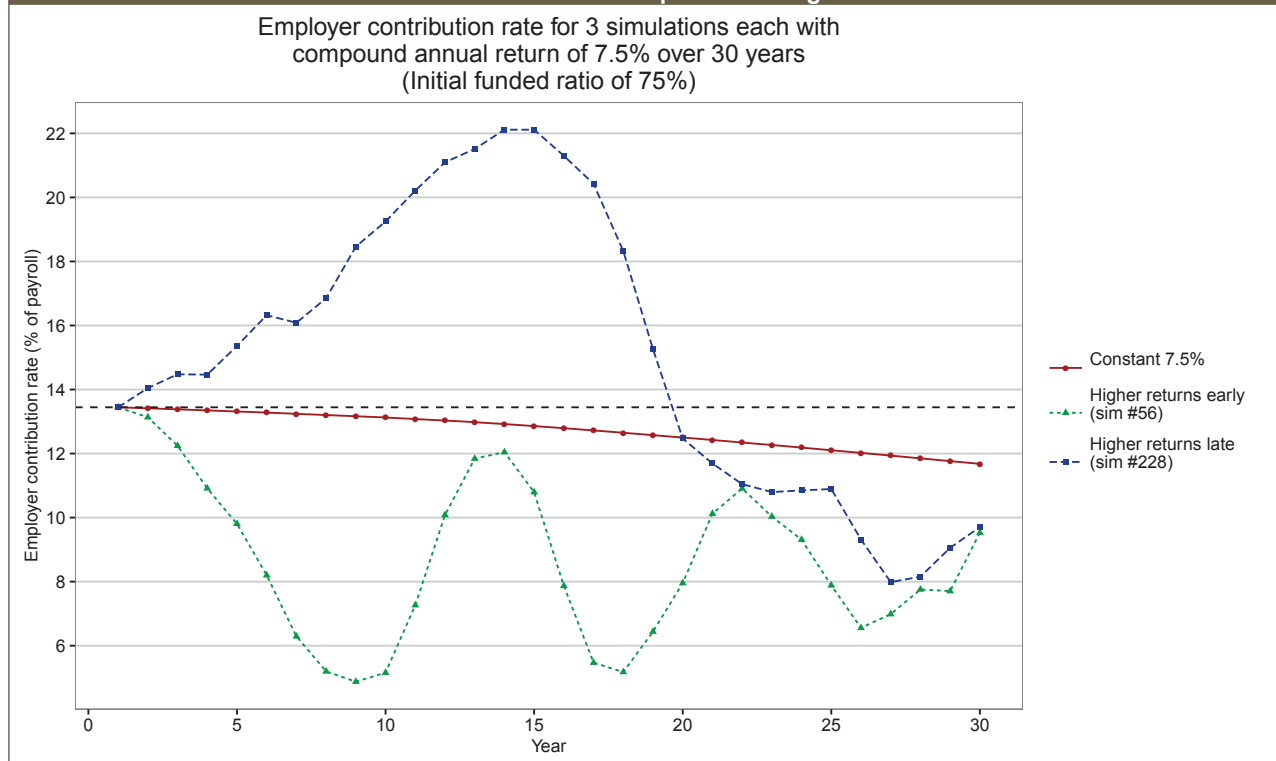
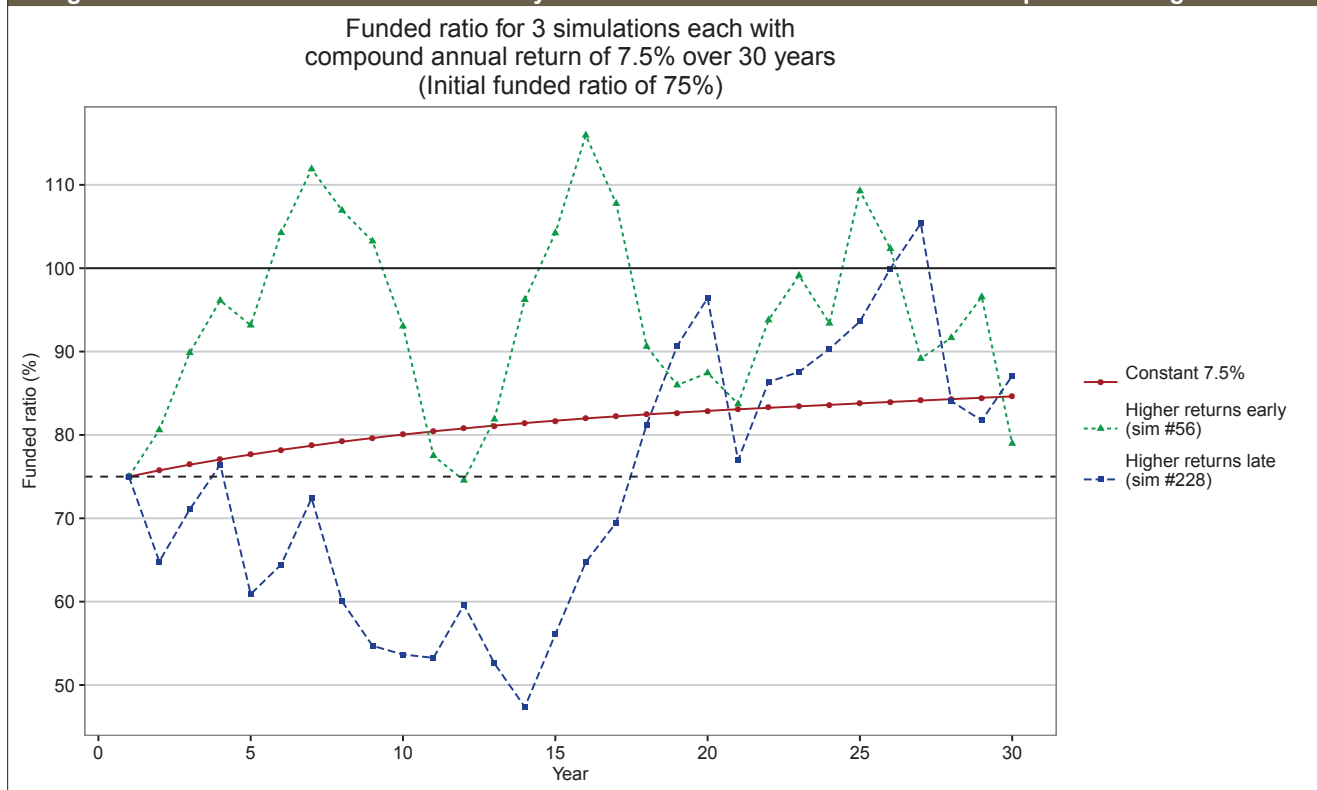


Figure 4 shows employer contribution rates for the three simulations. The rates are quite stable for the run in which returns are precisely 7.5 percent every year — the typical actuarial assumption — but they are highly variable for the other two simulation runs. Simulation #228 with low early returns and high later returns results in substantial but varying contribution rates, increasing by about 8 percentage points of payroll in the first fifteen years (a 57 percent increase), then falling almost continuously as higher investment returns are achieved. Simulation #56, with high early returns and low later returns, generates lower contributions than the deterministic simulation throughout the first thirty years, but contributions fluctuate considerably, rising or falling by as much as 6 percentage points of payroll in periods of three to four years.

Thus, even if a plan achieves its assumed returns over a thirty-year period, contributions — and legislators who must adjust government budgets to accommodate contributions — will be on a bumpy ride.

Figure 5 shows the variability in funded status. After thirty years the plan remains less than 90 percent funded in all three simulations, even though the plan achieved its investment return assumption and the employer made all actuarially determined contributions. In the higher-returns-early simulation (#56), funded status rises above 100 percent at several points during the first thirty years. In the higher-returns-late simulation (#228) the funded ratio falls below 50 percent at fourteen years but then rises nearly continually for about ten years before falling. It may seem

Figure 5. Funded Status Varies Dramatically for Simulations That Have the Same Compound Average Return



comforting to see that when the funded ratio dipped below 50 percent in year fourteen, future simulated investment returns combined with employer contribution increases were so great that they pulled funding up to nearly 100 percent over the next six years. But politicians who find themselves in such a situation would have no comfort: they would be faced with many years of high contributions and no reason to believe that future returns will be high simply because past returns were low. It happens by design in our simulation because the two simulations we selected were known to achieve assumed returns by the end of thirty years.

Other simulations can produce very different results, including funded status above 100 percent for extended periods, and funded status well below 40 percent, particularly in simulations that do not achieve a 7.5 percent compound annual return in the first thirty years.

It is not just the timing of investment returns and of employer contributions that varies across the three runs. The different simulations result in very different values for the cumulative present value of employer contributions, as shown in Table 2. (We calculate the present value of contributions and payroll using a 7.5 percent discount rate. Alternative discount rates would result in different present-value calculations.) In the higher-returns-early scenario (sim #56), the present value of employer contributions over the first thirty years is 9.1 percent of the present value of payroll. By contrast, the present value is 15.9 percent of payroll in the higher-returns-later scenario (sim #228). The present value for the

Table 2. Funded Ratios, Employer Contributions, and the Cumulative Present Value of Employer Contributions Vary Even When Compound Returns Are the Same

Three simulations with compound annual return of 7.5%			
	Constant 7.5%	Higher returns early (sim #56)	Higher returns later (sim #228)
Average compound annual return:			
Years 1-15	7.5%	10.3%	5.4%
Years 16-30	7.5%	4.8%	9.7%
Years 1-30	7.5%	7.5%	7.5%
Funded ratio, year 30	84.6%	79.0%	86.8%
Present value of employer contributions, years 1-30, as % of present value of payroll (discounted at 7.5%)	13.0	9.1	15.9
% by which present value of employer contributions is above (below) constant 7.5% scenario	0.0%	(30.0%)	21.8%

constant 7.5 percent scenario falls between the two extremes. Each of these variable-return simulations differs from the constant-returns simulation by at least 20 percent, and the higher-returns-later simulation is 75 percent more expensive than the higher-returns-early scenario. Thus, the order in which investment returns arrive is important, even when the compound annual return is the same — high returns early can have a large beneficial impact on overall contributions, while higher returns later can have a large detrimental impact.

What causes the order of returns to matter? Negative cash flow, before considering investment returns (i.e., benefit payouts that exceed total contributions), plays a role. When a plan has negative cash flow, relative to a plan that does not, investible assets will be higher in early years than later. In this case, good returns early will generate more investment income than good returns later, because they will be applied to greater investible assets. Required contributions therefore will be lower. The interplay between the order of returns, contributions, and plan funded ratio can become quite complex when contributions are smoothed; as a result, it is difficult to understand and predict precisely how contributions and funded status will vary. Most plans currently have negative cash flow before investment income and so these issues are quite relevant.

Summary Results

We now summarize results for different funding policies, each of which we simulated 1,000 times. In all cases, investment

returns have an expected compound annual return of 7.5 percent (consistent with the typical plan assumption) and a 12 percent standard deviation.

To keep graphs understandable, we focus on just four policies that we selected from the full set of simulated policies. We highlight these policies because three demonstrate the range of current practice and the fourth represents a proposed reform. The four policies and their labeling are:

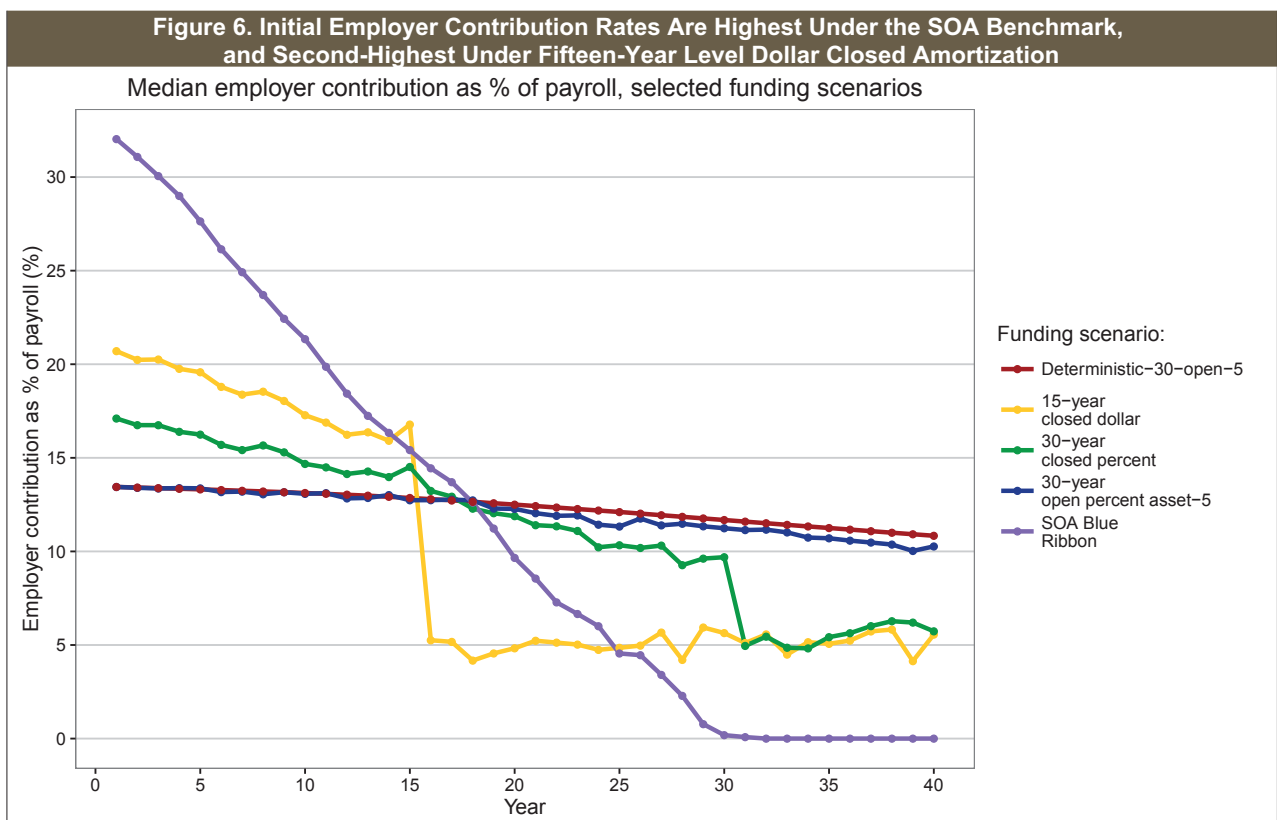
- **Fifteen-year closed dollar:** Gains and losses are amortized over a fifteen-year closed period, in level-dollar amounts (similar to a fixed-rate home mortgage). This approach repays gains and losses completely after fifteen years. Ten plans used a similar approach in 2013, although they generally combined it with asset smoothing.
- **Thirty-year closed percent:** Gains and losses are amortized over a thirty-year closed period, as a level percentage of payroll, with no asset smoothing. Early payments are lower than later payments, with annual payments rising at the rate of payroll growth (typically 3 to 4 percent per year). This method repays gains and losses completely at the end of thirty years, with the repayment occurring disproportionately in later years. Fourteen plans used a similar approach in 2013, generally combined with asset smoothing.
- **Thirty-year open percent asset-5:** Gains and losses are amortized over a thirty-year open period as a level percentage of payroll, with five-year asset smoothing. As discussed earlier, open amortization means that the amortization period is constantly reset so that there is always a new thirty-year period. The liability therefore is never fully repaid, although it is reduced, and can decline substantially as a percentage of payroll. Twenty-eight plans accounting for well over 40 percent of the unfunded liabilities of large plans use a similar method.
- **SOA Blue Ribbon:** As described earlier and as detailed in the appendix, this is the contribution benchmark policy that was proposed by a panel commissioned by the Society of Actuaries. The panel proposed that pension funds compare their actuarially determined contributions with contributions determined under this approach. Plan liabilities and contributions would be calculated using a forward-looking market-based discount rate that is 5.9 percent in our simulations. The benchmark uses fifteen-year level-percent open amortization, with five-year asset smoothing.

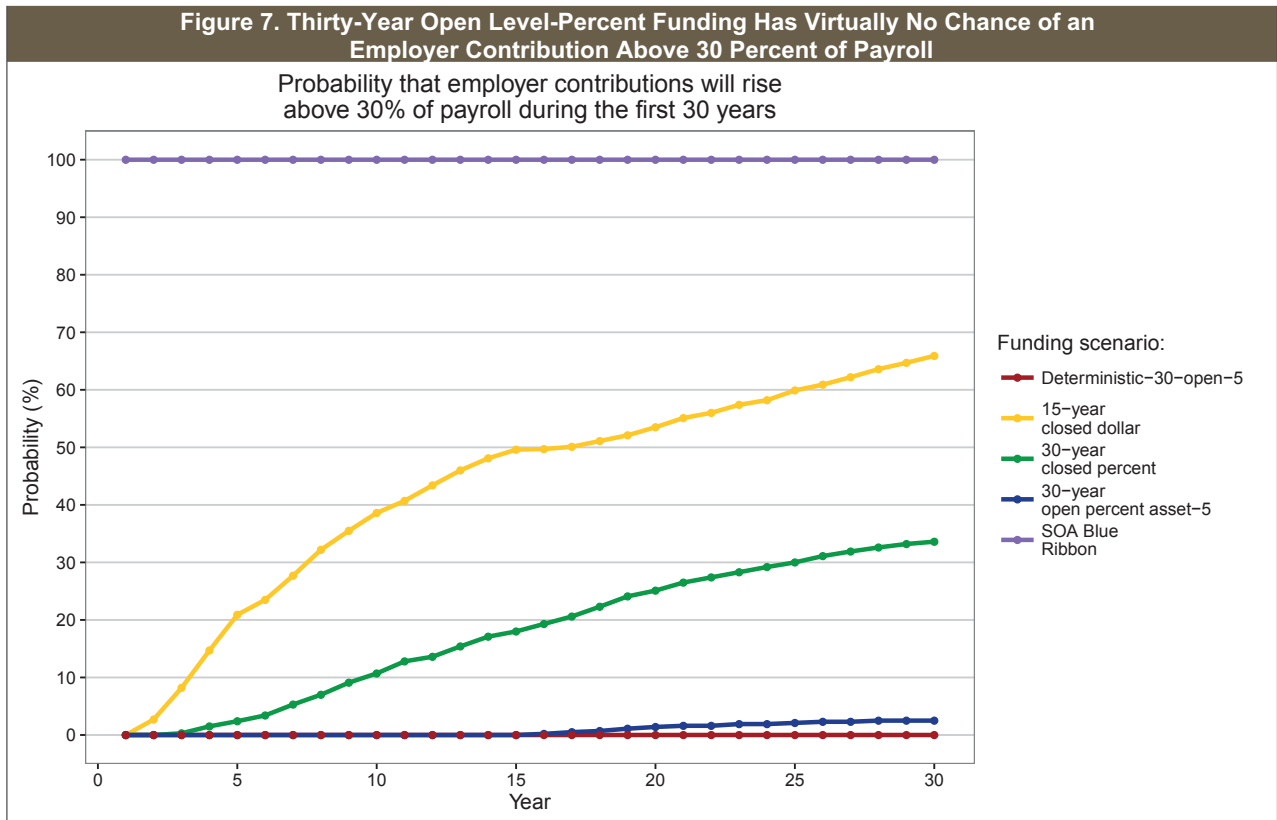
We also show the deterministic scenario (7.5 percent investment return each and every year) for the thirty-year open level-percent funding policy with five-year asset smoothing, labeled as **Deterministic-30-open-5**.

Employer Contributions

We begin with the median employer contribution rate across 1,000 simulations under each of the four policies (Figure 6). We show results for forty years so that the declines in contributions after closed amortization periods end are apparent:

- The initial contribution under the Society of Actuaries (SOA) benchmark is as much as twice that of the other policies, primarily because the market-based discount rate is 5.9 percent, compared with the 7.5 percent assumed for all of the other funding policies. In the simulations the plan earns 7.5 percent on average, despite the lower discount rate, and so the contribution falls steadily.³¹ It is not surprising that the SOA Benchmark has a salutary impact on funded ratios, and that governments might prefer other methods that have much lower initial contributions.
- Employer contributions fall sharply in the fifteen-year and thirty-year closed amortization policies, after their respective amortization periods end, because the initial 25 percent underfunding is treated as a shock that occurred in a single event and it is fully amortized at the end of the period.
- The thirty-year open policy has the lowest contributions between years two and fifteen, and the highest contributions after year thirty.

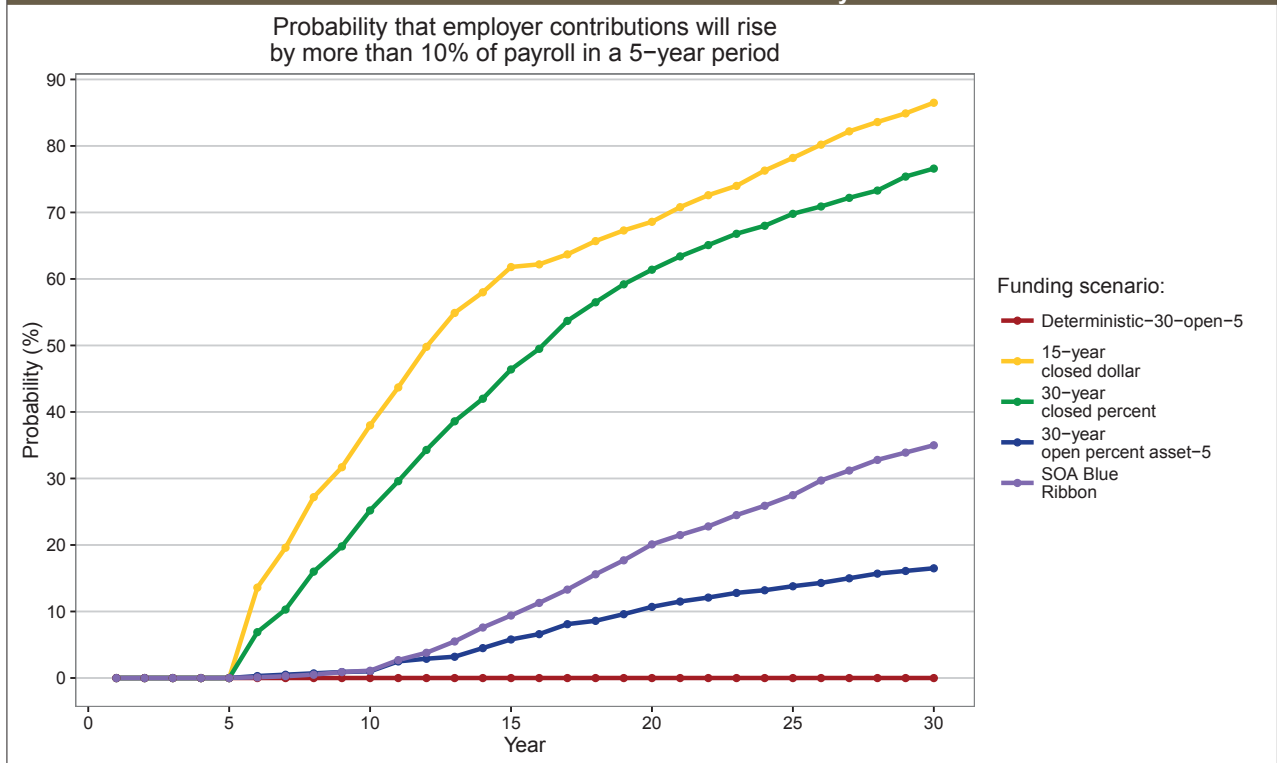




While medians show what happens in the typical case, they don't reveal the implications of investment return volatility. For that, we need the probabilistic measures described earlier. Figure 7 shows the probability that employer contributions will rise above 30 percent of payroll during the first thirty years. The deterministic scenario, which has an employer contribution ranging from about 14 percent of payroll to 12 percent depending on the year (see Figure 6 above), has zero chance of the employer contribution rising above 30 percent. Perhaps surprisingly, the highly smoothed policy of thirty-year open funding as a level percentage of payroll, with five-year asset smoothing, also has near-zero probability of employer contributions rising above 30 percent. At the other extreme, the SOA benchmark would start out with contributions above 30 percent and so the probability is 100 percent. The two closed-period policies fall between these extremes.

Figure 8 shows the probability that employer contributions will rise by more than 10 percent of payroll in any five-year period within the first thirty years. There is no chance of this when investment returns are deterministic – 7.5 percent each and every year. However, it is possible when investment returns vary, in which case investment shortfalls may trigger contribution increases. The quicker and more forcefully that a funding policy responds to investment shortfalls, the greater the probability that contributions will rise by more than 10 percent. Thus, this probability is greatest for the fifteen-year closed period level-dollar funding policy: it repays shortfalls over a fixed relatively short

Figure 8. Closed Funding Methods Have a Greater Chance of Contribution Increases of More Than 10 Percent of Payroll

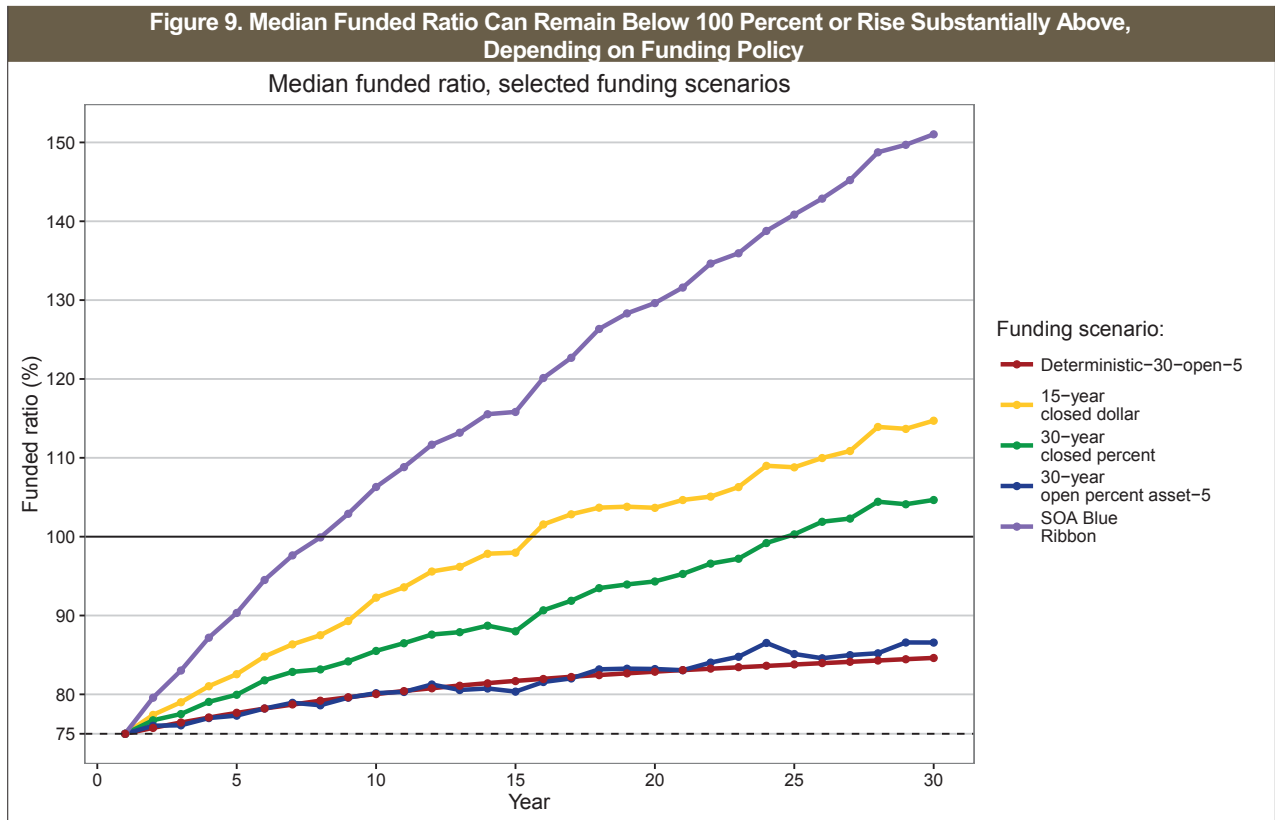


period, with constant-dollar payments that are higher than under level-percent funding policies.

It is easy to see why the very stretched-out policy of thirty-year open-period funding as a level percentage of payroll is attractive to employers. Unlike other policies currently used by funds, it has near-zero chance of employer contributions rising above 30 percent, and a very low probability that contributions will rise by more than 10 percentage points in a five-year period. It provides stability to plan sponsors. That stability comes at a price, however: a risk of severe underfunding when investment returns vary from year to year.

Impact on Plan Funding

Figure 9 shows the median funded ratio across 1,000 simulations for each of the funding policies. Under the deterministic scenario and its stochastic counterpart (thirty-year open level-percent funding with five-year asset smoothing), the funded ratio rises from the initial 75 percent to about 87 percent after thirty years: The plan never quite achieves full funding but moves in that direction. The two closed-period methods eventually raise the funded ratio above 100 percent, in part because contributions are not allowed to become negative (the sponsor cannot withdraw money from the plan if it becomes overfunded, so there is always upward pressure on funding). The SOA benchmark, as we have modeled it, overfunds the plan substantially – it discounts



liabilities at 5.9 percent,³² but then achieves compound investment returns of 7.5 percent.

As with employer contributions, medians do not provide information on the risk of underfunding. Figure 10 shows the probability that the funded ratio will fall below 40 percent during the first thirty years. Here the results are the opposite of the results for employer contributions: The funding policies that repay shortfalls most aggressively and quickly have the least chance of a funded ratio falling below 40 percent. However, the relatively common funding policy of thirty-year open level-percent funding with five-year asset smoothing has a far greater risk of severe underfunding, reaching 17 percent by the end of thirty years — a one in six chance of severe underfunding.

Figure 10 demonstrates a very important conclusion: When the assumed investment return is achieved each and every year and actuarial contributions are paid, as in the deterministic scenario, there is zero risk of severe underfunding. *But if investment returns vary from year to year, there can be a substantial risk of severe underfunding (a one in six chance in this case), even if expected returns are correct on average and even if governments pay full actuarial contributions.*

Figure 11 shows the risk of a 75-percent funded plan falling below three different funded-ratio thresholds when the common funding policy is used: 60 percent, 50 percent, and 40 percent. The red line shows the likelihood of falling below 60 percent: there is a

Figure 10. Thirty-Year Open Level-Percent Funding Runs Far Greater Risks of Severe Underfunding Than the Other Funding Policies

Probability that the funded ratio will fall below 40% during the first 30 years

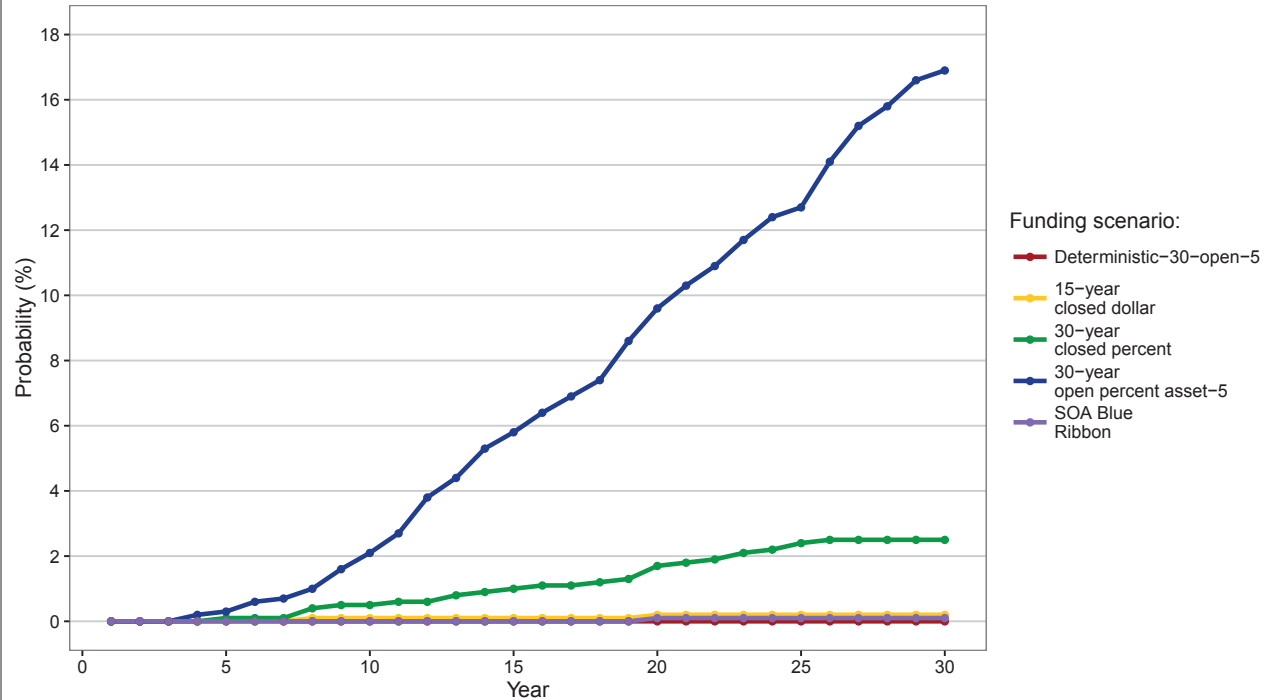
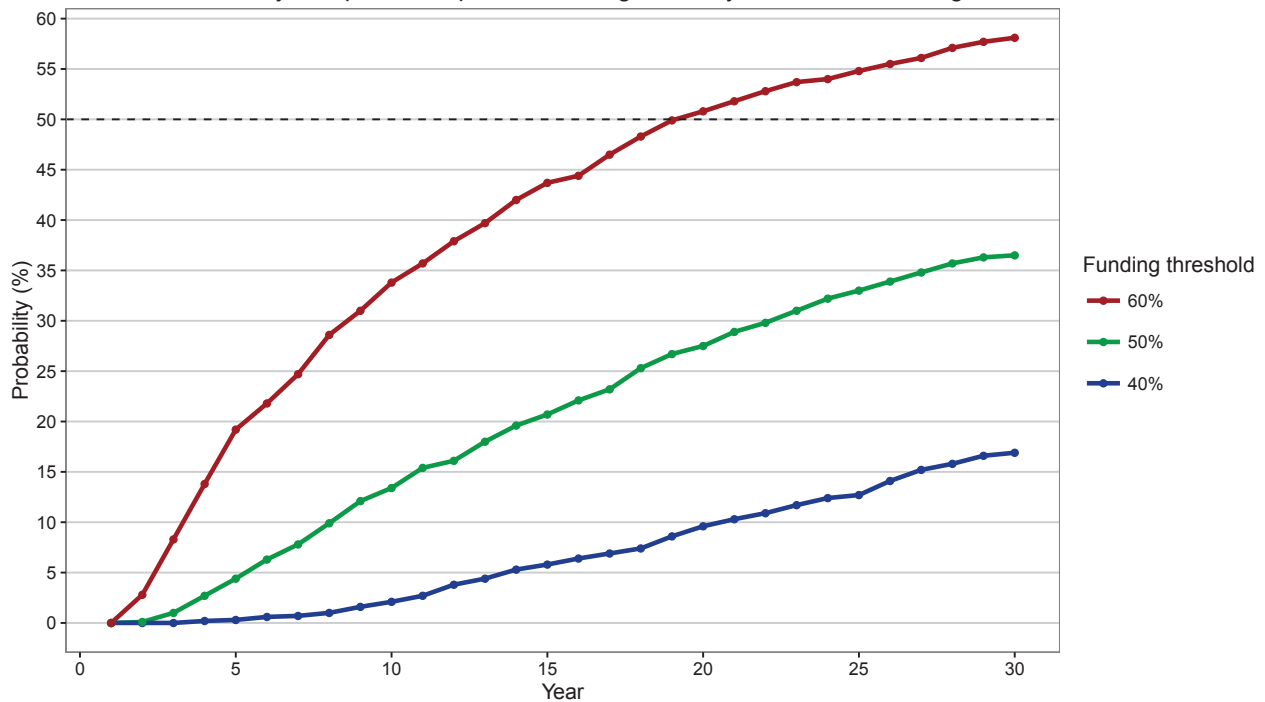


Figure 11. A Typical Plan That Starts Out 75 Percent Funded and Uses a Common Funding Policy Has More Than a 50 Percent Chance of Falling Below 60 Percent Funding Within Twenty Years

Probability that the funded ratio will fall below a given threshold during the first 30 years under 30-year open level-percent funding, with 5-year asset smoothing

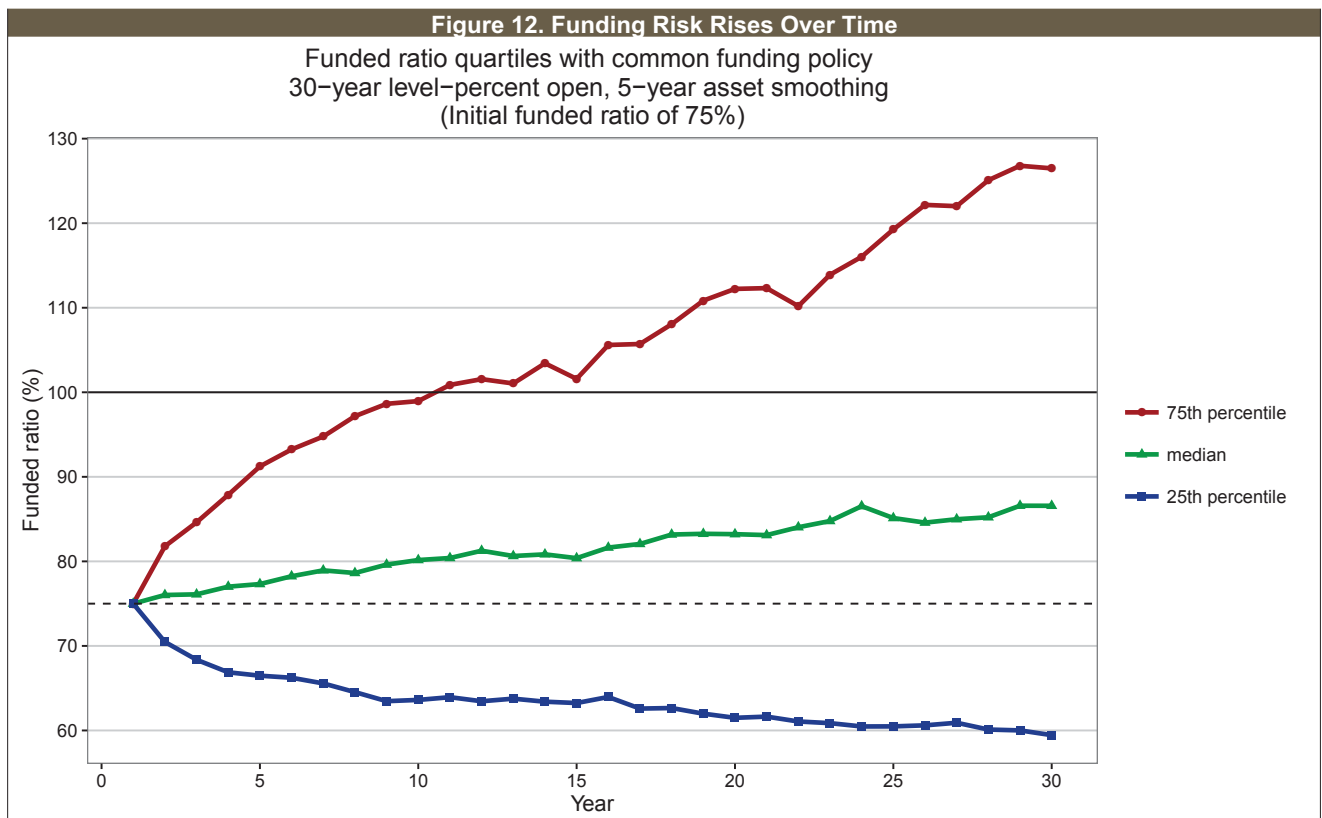


more than 50 percent chance of falling below this level within twenty years.

The three elements of the amortization method affect the degree to which costs are pushed to the future. The amortization period and the level-percent versus level-dollar choice affect each other. Extending the amortization period from fifteen years to thirty years has a bigger negative impact on funded status when combined with the level-percent method than when combined with the level dollar method. And switching to level percent from level dollar has a bigger negative impact on funded status when combined with thirty-year amortization than with a fifteen-year period. Replacing open amortization with closed amortization can considerably improve the long-run funded status at the expense of higher contributions.

None of the smoothing policies can prevent sharp rises or falls in contributions over the long run. If funds are invested in risky assets, smoothing policies can alter the time it takes to reach higher contributions, but does not change the long-run path markedly.

Finally, Figure 12, which shows quartiles for the funded ratio of the pension plan over time under the typical smoothing policy, illustrates a conclusion that may be surprising to some people. The risk of better than expected or worse than expected funding outcomes increases over time. That is, the gap between the 75th percentile and the 25th percentile grows over time. This results primarily because investment returns in the simulations are



independent from year to year. When that is true, which is often assumed in financial simulations, it is well known that the volatility of asset values rises over time. While the variability in *compound annual returns* diminishes over time, this is more than offset by compounding those returns over more years. If returns are not independent — if, for example, bad investment years are followed by good years, and vice versa — the gap in Figure 12 might not widen so significantly, or conceivably it could even narrow. Under some investment return assumptions, the gap could widen even more.

Figure 12 also shows that the risk is skewed in the sense that the best outcomes — high funded ratios — rise by more than the worst outcomes fall. The lower bound of the funded ratio is zero, but there is no limit to how high the funded ratio can rise. In addition, because the model does not allow negative contributions when the plan becomes overfunded, there is a built-in tendency for further overfunding. This is consistent with the practice of plan funding — sponsors generally are not allowed to withdraw funds if a plan becomes overfunded.

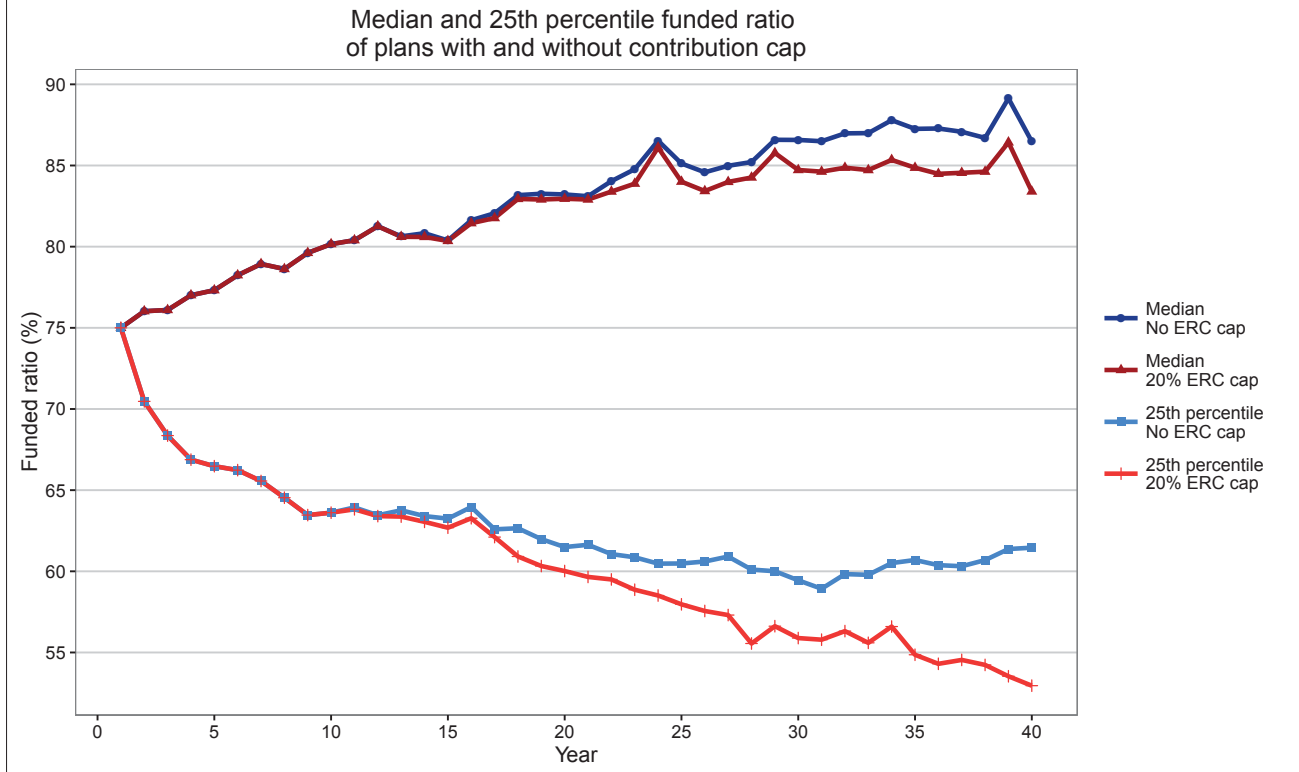
What Happens If Contributions Are Less Than Actuarially Determined Contributions?

Our analysis up to this point assumes that plan sponsors always make full required contributions. In practice, this often is not true, especially when the required contribution is so high that it is difficult for the sponsor to afford. Andrew Biggs of the American Enterprise Institute found that the higher the actuarially required contribution (ARC) is as a percentage of payroll, the less likely the sponsor is to pay the full ARC and the lower the percentage of ARC is paid.³³

We model the consequences of a shortfall in paying the actuarially determined contribution by imposing a cap on the employer contribution as percentage of payroll, an approach similar to that used by Biggs, in essence assuming that employer underpayment occurs at this point. We model a cap on the employer contribution rate that is approximately three times the “steady state” employer contribution rate of a fully funded plan that achieves a constant investment return equal to its assumed return. For a plan with the common funding policy of thirty-year level-percent open amortization with five-year asset smoothing and a 7.5 percent discount rate, the “steady state” employer contribution rate is around 6.8 percent, and so we model an employer contribution cap of 20 percent of payroll (about three times 6.8 percent).

Figure 13 shows the impact of this underpayment behavior on the plan’s funded status. The two lines on the upper part of the figure give the median funded ratio of the plan with and without the contribution cap. The cap has little impact on the median funded ratio until year twenty, and in year forty the funded ratio of the plan with contribution cap is about 3 percent lower than the one without the cap. The impact is relatively small because the

Figure 13. Contribution Underpayment (Modeled as a Cap On Employer Contributions) Has a Larger Impact on Funded Status Under Bad Return Scenarios (e.g., at the 25th Percentile) Than Under Better Scenarios (e.g., at the Median)



median contribution rate is lower than 15 percent (see Figure 6), which is well below the 20 percent contribution cap. However, the effect of the contribution cap is more prominent when the plan faces bad return scenarios and the contribution cap is therefore triggered more frequently, which is represented by the 25th percentile lines of funded ratio on the lower part of the graph (25 percent of 1,000 simulations have funded ratios lower than those presented by the 25th percentile lines). In year forty, the 25th percentile funded ratio of the capped plan is almost 10 percentage points lower than the uncapped plan. In this simulation result, the contribution cap is not triggered a lot in early years because of the low initial employer contribution rate. If the initial contribution rate is higher, the cap would be triggered more frequently, leading to a larger negative impact on the plan's long-term funded status.

The Trade-Off Between Contribution Volatility and the Risk of Underfunding

As much of the previous discussion suggests, there is a trade-off between contribution volatility and the risk of underfunding. Governments understandably want stable contributions – inherently difficult to achieve when investment returns are volatile. In general, the more that plans and governments rely on amortization and other smoothing techniques to reduce contribution volatility, the greater the risk that funding ratios will fall to low levels

– perhaps to levels that are not politically sustainable and that lead to significant benefit cuts or risk of nonpayment.

Figure 14 illustrates this tradeoff. The vertical axis shows the probability that the funded ratio will fall below 40 percent at any point during the first thirty years. The horizontal axis shows a measure of contribution volatility – the probability that employer contributions will rise by more than 10 percent of payroll during any five years in the first thirty. The graph makes clear that plans that use funding policies with low contribution volatility, such as thirty-year level-percent open with five-year asset smoothing, have much greater likelihood of encountering a very low funded ratio.

Finally, Table 3 shows the probability of severe underfunding and the contribution volatility measures for a larger group of funding policies, including all of the policies in Figure 14.

Conclusions and Policy Implications

The most-common funding policies and practices reduce contribution volatility at the same time that they increase the likelihood of severe underfunding. These policies are unlikely to bring underfunded plans to full funding within thirty years, even if investment-return assumptions are met every single year and employers make full actuarially determined contributions. For example, a 75 percent funded plan using a common policy of

Figure 14. Contribution Smoothing Policies Reduce Contribution Volatility But Increase the Risk of Severe Underfunding

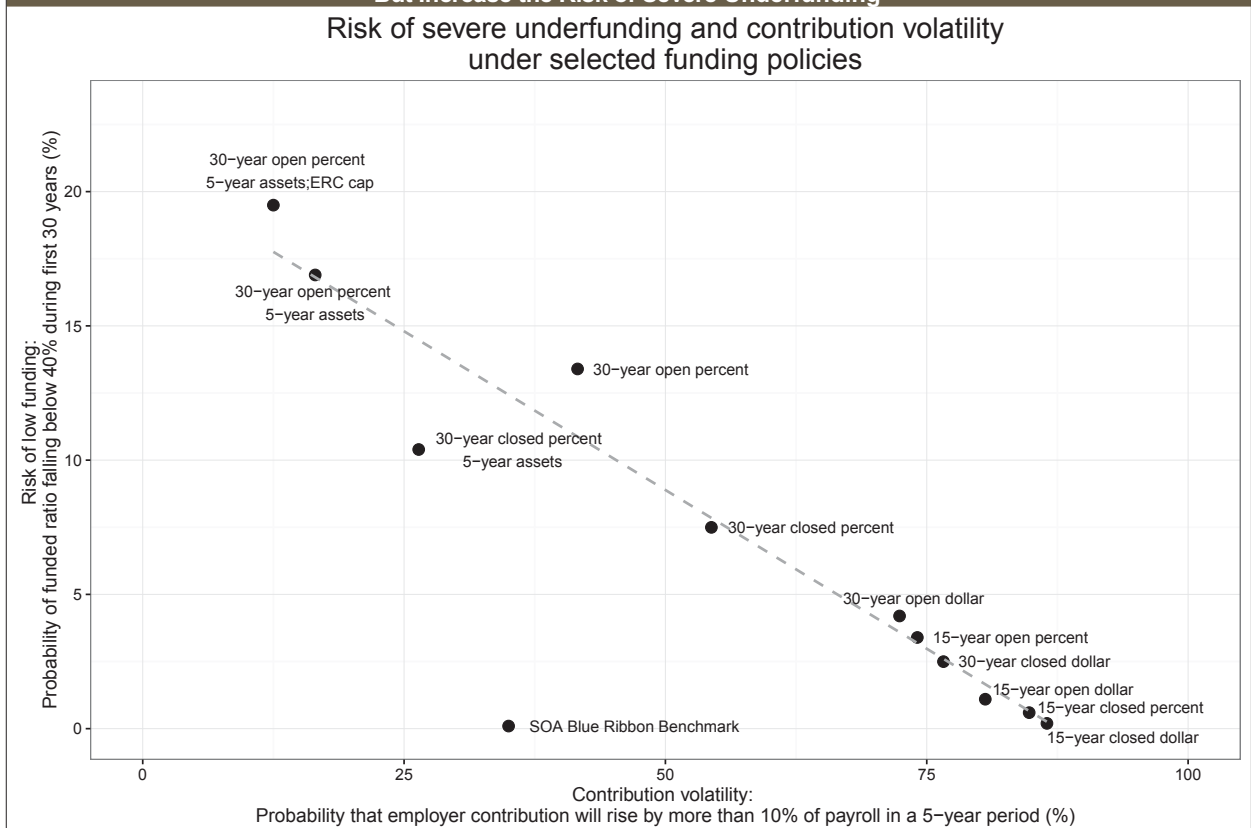


Table 3. Funded Ratio Risk and Contribution Volatility for Selected Funding Policies

Funding Policy	Probability that funded ratio will fall below 40% during first 30 years	Probability that employer contributions will rise above 30% of payroll during the first 30 years	Probability that employer contributions will rise by more than 10% of payroll in a 5-year period
15-year level dollar - closed	0.2%	65.9%	86.5%
15-year level percent - closed	0.6%	60.5%	84.8%
30-year level dollar - closed	2.5%	33.6%	76.6%
30-year level percent - closed	7.5%	22.2%	54.4%
30-year level percent - closed; 5-year asset smoothing	10.4%	19.3%	26.4%
15-year level dollar - open	1.1%	48.7%	80.6%
15-year level percent - open	3.4%	29.5%	74.1%
30-year level dollar - open	4.2%	24.8%	72.4%
30-year level percent - open	13.4%	3.5%	41.6%
30-year level percent - open; 5-year asset smoothing	16.9%	2.5%	16.5%
30-year level percent - closed; 5-year asset smoothing; 20% ERC cap	19.5%	0.0%	12.5%
SOA Blue Ribbon Panel Benchmark	0.1%	100.0%	35.0%

Note: Funding policy in the SOA Blue Ribbon Panel Benchmark consists of 15-year level-percent open amortization with 5-year asset smoothing. We have used a 5.9% discount rate for liability calculations, based on our analysis of current market conditions. However, the pension fund is assumed to earn a long-run compound return of 7.5%, to maintain comparability with other funding policies in the table.

thirty-year open amortization as a constant percentage of payroll, with five-year asset smoothing, would only reach 85 percent funding after thirty years even if it earned 7.5 percent every year. When investment returns are variable, plans and their sponsors face substantial risk of potential crises: The same plan would face a one in six chance of falling below 40 percent funding within thirty years if its investment return assumption is correct on average but has a 12 percent standard deviation. If sponsors do not pay full actuarial contributions, or if reasonable expected returns are less than 7.5 percent, the risk of severe underfunding would be greater.

This raises important questions about the impact that pension contributions will have on state and local government taxes and spending, and questions about the security of pension benefits. When returns are good and contributions are driven downward, will governments resist the urge to increase benefits, or will they raise them as some have done in the past? Will they resist the urge to embark on other spending programs that may have to be cut in later years if returns are bad? When returns are bad and requested contributions are driven up sharply, will elected officials be willing to raise taxes and cut spending to support those contributions, or will many pay less than actuarially determined amounts as often has occurred previously?

It is clear that pension funding policy is not adequate in the current environment. In the current low-interest-rate and low-inflation environment plans are taking the risk of substantial investment income shortfalls to have reasonable chances of achieving their investment return assumptions. This means that investment earnings will be particularly volatile and funding policy takes on greater importance. In the face of earnings volatility, plans can attempt to dampen contribution volatility through the smoothing methods available to them, but only at the expense of making funded ratios more volatile, increasing the risk that pension funds will become severely underfunded and that required contribution increases will be politically untenable.

There is no easy way out. Pension plans can de-risk, reducing the volatility of their investment returns and reducing the volatility of contributions. However, reducing risk almost certainly will require lowering earnings assumptions, which will drive up contribution demands from governments and crowd out services or require tax increases.

This is the first report based upon the Rockefeller Institute of Government's pension simulation model. Future reports will examine other aspects of the risks faced and taken by public pension plans, and the implications for state and local governments, their taxpayers, and those who benefit from government services and investments.

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Appendix

Arithmetic and Geometric Mean Investment Returns

When investment returns are variable, compound average returns (also known as geometric mean returns) will be lower than the arithmetic average of returns. For example, if starting assets of \$100 earn a return of 20 percent in year one followed by a return of 0 percent in year two, then ending assets will be \$120. This is a compound average return of approximately 9.545 percent (100×1.09545^2 is approximately 120), which is lower than the arithmetic average of 10 percent. This phenomenon is sometimes called volatility drag. A commonly used approximation, which we use in this analysis, is that the long-run compound average return is approximately equal to the arithmetic mean minus one-half of the variance. (The accuracy of this approximation depends on the actual distribution of returns over time.) Thus, with a 12 percent standard deviation, a 7.5 percent long-run compound average return requires approximately an 8.22 percent arithmetic mean return ($0.0822 - (0.12^2)/2 \approx .075$). The volatility drag is seventy-two basis points – the difference between 8.22 percent and 7.50 percent. In other words, for a pension fund to earn a long-run compound average return of 7.5 percent, it needs to earn an arithmetic mean return of 8.22 percent if the standard deviation is 12 percent. Thus, in our model we draw random returns from a normal distribution with an 8.22 percent arithmetic mean and a 12 percent standard deviation.

Actuarial practice does not appear to be clear or uniform on whether plans consider the arithmetic mean or the geometric mean of expected investment returns. Some pension funds appear to take volatility drag into account, so that if they assume long-run returns of 7.5 percent, it is based on an analysis of market conditions that leads them to believe that the average return in any given year might be 8.2 percent or so. Other funds that assume long-run returns of 7.5 percent appear to reach this conclusion based on an analysis of market conditions that leads them to believe that the average return in any given year might be 7.5 percent. The Blue Ribbon Panel of the Society of Actuaries remarked on this and proposed that actuaries base their analysis of returns on geometric mean (i.e., compound average) returns.³⁴

The SOA Blue Ribbon Panel's Standardized Contribution Benchmark

In 2014 a panel commissioned by the Society of Actuaries proposed that public pension plans disclose (but not necessarily fund) a standardized contribution, to help users assess the adequacy and reasonableness of the plan's actual or recommended contributions.³⁵ The Standardized Contribution Benchmark (SCB) would have the following key features:

- The discount rate would continue to be based upon an earnings assumption as is current actuarial practice. How-

ever, it would be “forward looking” and based upon current market conditions, rather than being based primarily on historical returns. The example contribution provided by the Blue Ribbon Panel in its 2014 report estimated a discount rate of 6.4 percent. In current market conditions, the equivalent rate would be about a half percent lower.

- Unfunded liabilities would be amortized using a fifteen-year open level-percent method.
- Five-year asset smoothing would be used.

Inflation and salary assumptions would be adjusted to be consistent with the discount rate assumptions.

These features generally would result in a higher benchmark contribution than contributions resulting from other methods for two reasons. First, the discount rate generally would be lower in the current market environment, resulting in (a) higher normal costs, and (b) higher actuarial liabilities and therefore higher unfunded liabilities to amortize. (These movements could be offset partially by lower inflation and salary assumptions.) Second, the fifteen-year amortization period is shorter than periods in common use (as Table 1 shows, 90 percent of plans use periods of sixteen years or more) and thus fifteen years usually would result in a higher contribution.

Endnotes

1. "2014 Annual Survey of Public Pensions: State-Administered Defined Benefit Data," Washington, DC: U.S. Census Bureau, 2014, <https://www.census.gov/govs/retire/>.
2. These estimates of unfunded liabilities, prepared by the U.S. Bureau of Economic Analysis and the Federal Reserve Board, are greater than those of actuaries. See Donald J. Boyd and Yimeng Yin, *By The Numbers Brief: State and Local Government Unfunded Pension Liabilities Rise by \$268 Billion in the Third Quarter of 2015* (Albany: Rockefeller Institute of Government, January 2016), http://www.rockinst.org/pdf/government_finance/2016-0120_By_the_Numbers_Brief_No4.pdf.
3. *Objectives and Principles for Funding Public Sector Pension Plans*, Issue Brief (Washington, DC: American Academy of Actuaries, February 2014), http://www.nasra.org/files/Topical%20Reports/Funding%20Policies/Public-Plans_IB-Funding-Policy_02-18-2014.pdf.
4. Henning Bohn, "Should Public Retirement Plans Be Fully Funded?," *Journal of Pension Economics and Finance* 10, 2 (2011): 195-219.
5. *Report of the Blue Ribbon Panel on Public Pension Plan Funding* (Schaumburg: Society of Actuaries, February 2014), <https://www.soa.org/blueribbonpanel/>.
6. Jeffrey R. Brown and David W. Wilcox, "Discounting State and Local Pension Liabilities," *American Economic Review* 99, 2 (May 2009).
7. *Report of the Blue Ribbon Panel on Public Pension Plan Funding*.
8. Alicia H. Munnell and Jean-Pierre Aubry, *The Funding of State and Local Pensions: 2014-2018* (Washington, DC: Center for State & Local Government Excellence, June 2015), http://slge.org/wp-content/uploads/2015/06/15-07-SLGE-Funding-of-State-and-Local-Pensions-2014-2018_web-final.pdf.
9. In the figure, the Treasury yield is the ten-year constant maturity yield, averaged over the typical public pension plan fiscal year (ending in June) from the daily rate available as variable DGS10 from the Federal Reserve Economic Data (FRED) website of the Federal Reserve Bank of St. Louis, <https://research.stlouisfed.org/fred2/>. The assumed investment returns are from several sources: (1) 2001-14 values are the unweighted mean of assumed returns, computed by the authors from Public Plans Data. 2001-14. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators, <http://crr.bc.edu/data/public-plans-database/>; (2) 1990-92, 1994, 1996, 1998, and 2000 are from Surveys of State and Local Government Employee Retirement Systems, generally authored by Paul Zorn and generally available through <https://www.questia.com/magazine/1G1-14379961/surveys-of-state-and-local-government-employee-retirement>; (3) 1989 is from Olivia S. Mitchell and Robert S. Smith, "Pension Funding in the Public Sector," *The Review of Economics and Statistics* 76, 2 (May 1994), 281, <http://www.uh.edu/~bsorensen/Mitchell%26SmithPensions.pdf>; and (4) 1975 is from *Pension Task Force Report On Public Employee Retirement Systems* (Washington, DC: House Committee on Education and Labor, 1978), 161, <https://babel.hathitrust.org/cgi/pt?id=pur1.32754076274483;view=1up;seq=6>. The graph is constructed to suggest that average assumed returns followed a fairly smooth path across the missing years from 1975 to 1989; based on inspection of data for a few plans that have published historical assumed returns, we believe this is a reasonable conclusion.
10. Robert Novy-Marx, and Joshua D. Rauh, "Public Pension Promises: How Big are They and What are They Worth?," *The Journal of Finance* 66, 4 (2011): 1211-49.
11. Consider a plan that provides a benefit of 2 percent per year of service times the average salary in the final five years of work, plus a 2 percent annual cost of living adjustment (COLA). For an employee who works from age twenty to sixty-five with 4 percent annual raises and lives until age eighty-five, the annual cost of benefits (the "normal cost") spread over salary earned during working years is 8.2 percent of payroll if the discount rate is 8 percent. However, as the discount rate falls, the annual cost rises. At a 6 percent rate, the normal cost would be 15.9 percent of payroll and at a 4 percent discount rate the cost would be 30 percent of payroll — nearly 4 times as great as at 8 percent. (Higher discount rates might be associated with higher price inflation and faster salary growth, affecting benefits and payroll. The example holds these constant.)

12. Line 6, "Imputed employer contributions," of "National Income and Products Account Table 7.24. Transactions of State and Local Government Defined Benefit Pension Plans," U.S. Department of Commerce, Bureau of Economic Analysis, <http://www.bea.gov/iTable/iTable.cfm?ReqID=9&step=1#reqid=9&step=3&isuri=1&903=395>. In addition, estimates of unfunded liability would be higher, potentially leading to greater amortization costs as well. (Note that the Bureau of Economic Analysis (BEA) calculates the cost of funding the accumulated benefit obligation rather than the projected benefit obligation, so these numbers are not exactly comparable to actuaries' estimates but are a useful approximation.)
13. In mid-February 2016 the yield on a 30-year Treasury bill was approximately 2.6 percent: <http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield>. Both the U.S. Bureau of Economic Analysis and Moody's Investors Service value liabilities using a high-quality corporate bond rate that was in the 4 to 4.5 percent range as of late 2015. Moody's uses the Citibank Pension Liability Index: <https://www.soa.org/professional-interests/pension/resources/pen-resources-pension.aspx> and <https://www.yieldbook.com/m/indices/citi-pension-liability.shtml>), an index of high credit-quality taxable bonds duration-weighted to be consistent with the liability duration of a typical private sector pension plan. This rate was 4.3% at January 31, 2016: <https://www.soa.org/Files/Xls/2015-09-pen-citigroup.xls> and https://www.yieldbook.com/x/ixFactSheet/CPLI_YC167.csv. "Adjustments to US State and Local Government Reported Pension Data," Moody's Investors Service, Viewed April 17, 2013, https://www.moodys.com/login.aspx?lang=en&cy=global&ReturnUrl=https%3a%2f%2fwww.moodys.com%2fviewresearchdoc.aspx%3fdocid%3dPBM_PBM151398%26lang%3den%26cy%3dglobal (Subscription Required). BEA used a rounded rate of 5.0 percent for analysis it did in relation to 2010 and 2011, when the CPLI was about a percentage point higher than in late 2015, so it appears broadly consistent with the Moody's approach. David G. Lenze, *State and Local Government Defined Benefit Pension Plans: Estimates of Liabilities and Employer Normal Costs by State, 2000-2011* (Washington, DC: Bureau of Economic Analysis, December 2013), <http://192.149.12.20/papers/pdf/State-and-Local-Government-Defined-Benefit-Pension-Plans.pdf>.
14. *Statement No. 25 of the Governmental Accounting Standards Board: Financial Reporting for Defined Benefit Pension Plans and Note Disclosures for Defined Contribution Plans* (Washington, DC: Governmental Accounting Standards Board, November 1994), <http://www.gasb.org/cs/BlobServer?blobkey=id&blobwhere=1175824062183&blobheader=application%252Fpdf&blobcol=urldata&blobtable=MungoBlobs>.
15. *Statement No. 27 of the Governmental Accounting Standards Board: Accounting for Pensions by State and Local Governmental Employers* (Washington, DC: Governmental Accounting Standards Board, November 1994), <http://www.gasb.org/resources/ccurl/44/286/GASBS-27.pdf>.
16. *Statement No. 67 of the Governmental Accounting Standards Board: Financial Reporting for Pension Plans, an Amendment of GASB Statement No. 25* (Washington, DC: Governmental Accounting Standards Board, June 2012), <http://gasb.org/resources/ccurl/399/602/GASBS67.pdf>.
17. *Statement No. 25 of the Governmental Accounting Standards Board*.
18. *LAGERS Forty-Fourth Comprehensive Annual Financial Report: Fiscal Year Ended June 30, 2012* (Jefferson City: Missouri Local Government Employees Retirement System, October 1, 2012), <http://www.molagers.org/assets/cafr-2012.pdf>.
19. Alicia H. Munnell, et al., *State and Local Pension Costs: Pre-Crisis, Post-Crisis, and Post-Reform* (Boston: Center for Retirement Research at Boston College, February 2013), http://crr.bc.edu/wp-content/uploads/2013/02/slp_30.pdf.
20. Because these are remaining amortization periods, they likely understate the intended amortization period of the typical plan. For example, a plan that uses thirty-year closed amortization that has already completed three years of amortization of an unfunded liability might report remaining amortization of twenty-seven years.
21. *Statement No. 25 of the Governmental Accounting Standards Board*.
22. Natalya Shnitser, "Funding Discipline for U.S. Public Pension Plans: An Empirical Analysis of Institutional Design," *Iowa Law Review* 100, 2 (2015): 663-713.

23. Each combination of a pension plan and a year is an observation in this context.
24. Raimond Maurer, Olivia S. Mitchell, and Ralph Rogalla, "Reforming the German Civil Servant Pension Plan," in *The Future of Public Employee Retirement Systems*, ed. Olivia S. Mitchell and Gary J. Anderson (Oxford and New York: Oxford University Press, 2009).
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26. Alicia H. Munnell, Jean-Pierre Aubry, and Josh Hurwitz, *How Sensitive Is Public Pension Funding to Investment Returns?* (Boston: Center for Retirement Research at Boston College, September 2013), http://crr.bc.edu/wp-content/uploads/2013/09/slp_34-508.pdf.
27. Andrew G. Biggs, "The Public Pension Quadrilemma: The Intersection of Investment Risk and Contribution Risk," *The Journal of Retirement* Summer 2, 1 (2014).
28. The 2.2 percent benefit factor is somewhat higher than the typical plan's factor, but the overall plan normal cost is consistent with typical plans in part because we do not model withdrawal of contributions or disability benefits.
29. CalPERS used a 12.96 percent standard deviation, Biggs assumed a 14 percent standard deviation, and Bonafede et al. estimated a 12.5 percent standard deviation. See *Annual Review of Funding Levels and Risks as of June 30, 2012* (Sacramento: California Public Employees' Retirement System, March 2013), <https://www.calpers.ca.gov/docs/forms-publications/annual-review-funding-2012.pdf>; Biggs, "The Public Pension Quadrilemma"; Julia K. Bonafede, Steven J. Foresti, and Russell J. Walker, *2015 Report on State Retirement Systems: Funding Levels and Asset Allocation* (Santa Monica: Wilshire Consulting, February 25, 2015), https://www.wilshire.com/media/38890/wilshire_2015_state_funding_report.pdf.
30. *Report of the Blue Ribbon Panel on Public Pension Plan Funding*.
31. The median SOA Benchmark contribution eventually falls to zero. This is because our model does not allow contributions to be negative, and so during periods of particularly good returns employers and employees contribute "too much" to the plan and eventually this can lead to extended periods in which employers do not have to make contributions.
32. For comparability with funded ratios for other policies, we compute the funded ratio under the SOA benchmark using a 7.5 percent discount rate.
33. Biggs, "The Public Pension Quadrilemma."
34. *Report of the Blue Ribbon Panel on Public Pension Plan Funding*.
35. Ibid.

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